Autoencoders and Variational Autoencoders

From compressing inputs to sampling a distribution

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ES 667 · Deep Learning · IIT Gandhinagar

Why reconstruction is not generation

The generative-modeling question

Q QUESTION

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today: build a generative model by **learning a latent space we can sample from**

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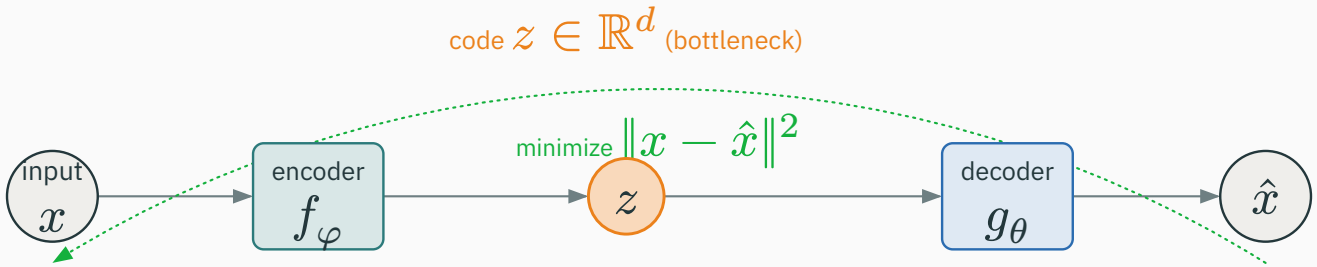

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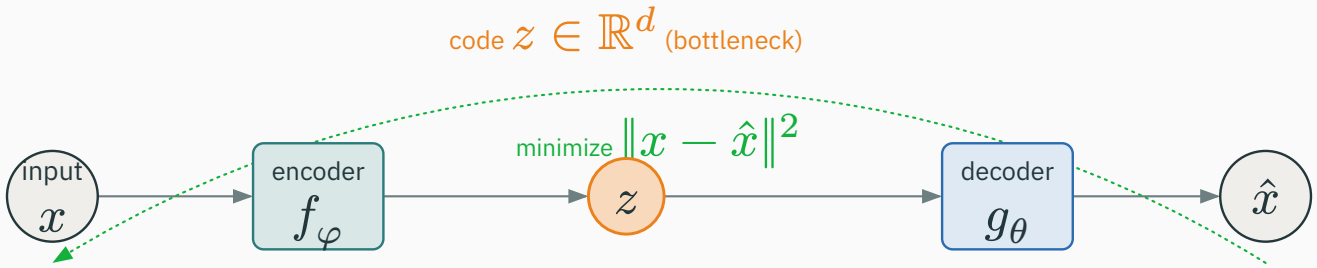

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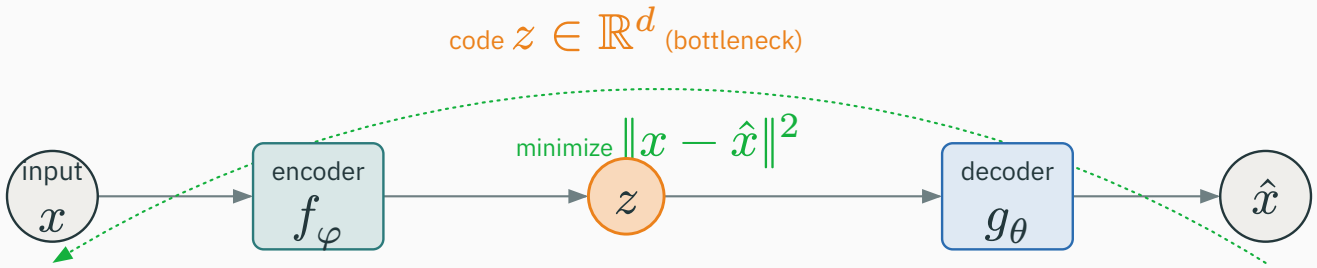

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(VAE convention: θ is the **generative / decoder** side, φ the encoder — the reverse of the classifier's $f_\theta \to g_\varphi$ in L18)

The bottleneck intuition

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If z is constrained (few dimensions), the network cannot store every input coordinate. To reconstruct well it must keep the **salient structure** and discard the rest.

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a wide-open code learns the **identity** map and learns nothing useful

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squared error = negative Gaussian log-likelihood — a first taste of the probabilistic view

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- **denoising** — corrupt the input, reconstruct the clean signal;
- **compression** — a compact code z for storage or transmission;
- **visualization** — a 2-D or 3-D code to plot the data;
- **features** — reuse z (or the encoder) for a downstream task.

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all of these use z as a **representation** — none of them require **sampling** new x

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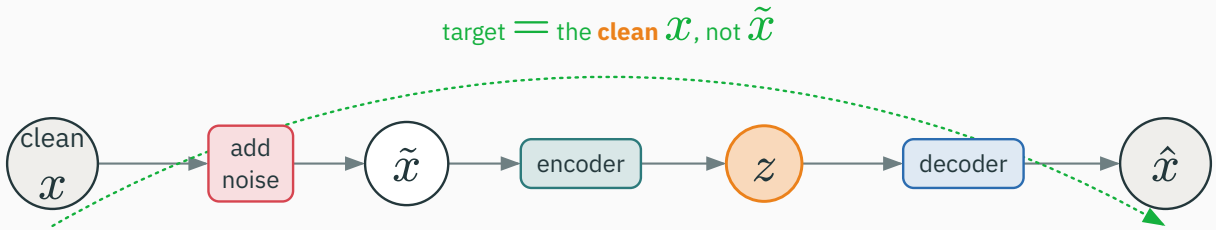

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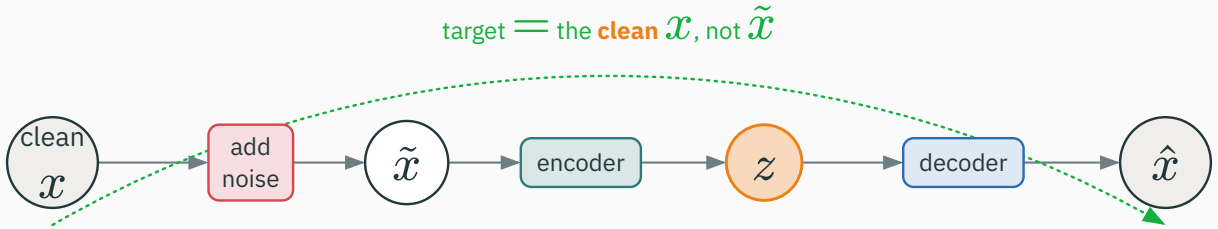

the target is the **clean** x; the encoder must learn what survives noise, not memorize pixels

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reconstruction says nothing about the **empty** parts of latent space

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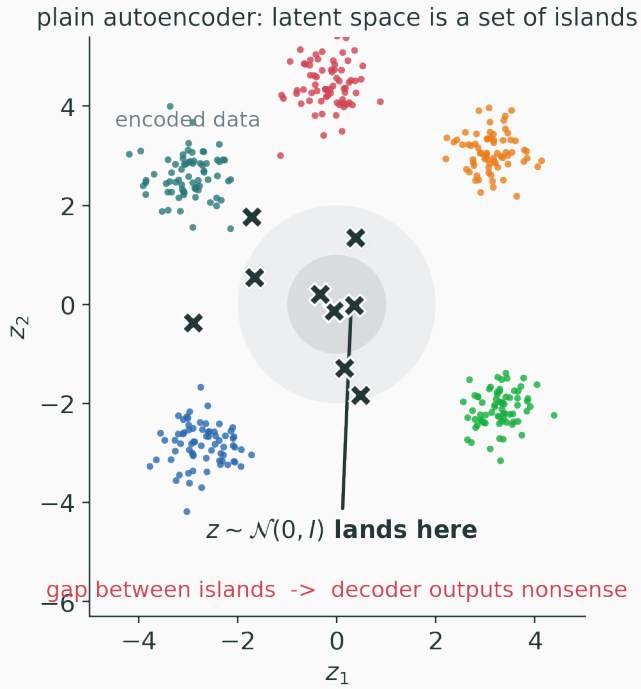

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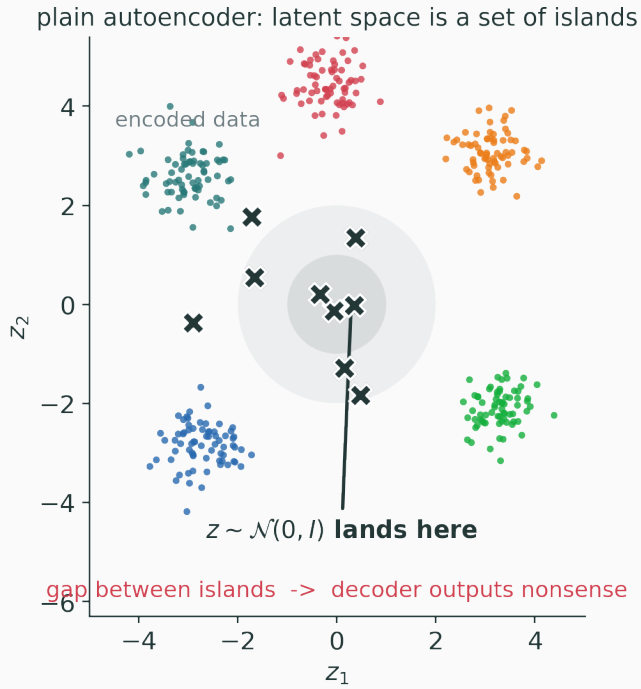

samples from $\mathcal{N}(0, I)$ mostly land in the **gap between islands** $\longrightarrow$ the decoder was never trained there

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We need two things at once: (1) a **known** distribution $p(z)$ we can sample, and (2) a decoder trained so that **every** likely z maps to something realistic. The VAE engineers exactly this.

Interactive: the latent space

I INTERACTIVE

INTERACTIVE

↗ nipunbatra.github.io/interactive-articles/vae-latent-explorer

Move a point around a 2-D latent space and watch the decoded output. See the islands of a plain autoencoder, and how prior samples fall into the gaps between them.

The VAE probabilistic model

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> the prior is **fixed and sampleable by construction** — that is what fixes the islands problem

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the familiar "reconstruction image" is just the conditional **mean** $g_\theta(z)$

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The evidence $p_\theta(x)$ integrates over **all** z through a nonlinear decoder. No closed form — so the posterior is out of reach too.

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The encoder now outputs a **distribution**, not a single point: a mean $\mu_\varphi(x)$ and a per-dimension variance $\sigma_\varphi(x)^2$. Each input maps to a small cloud in latent space.

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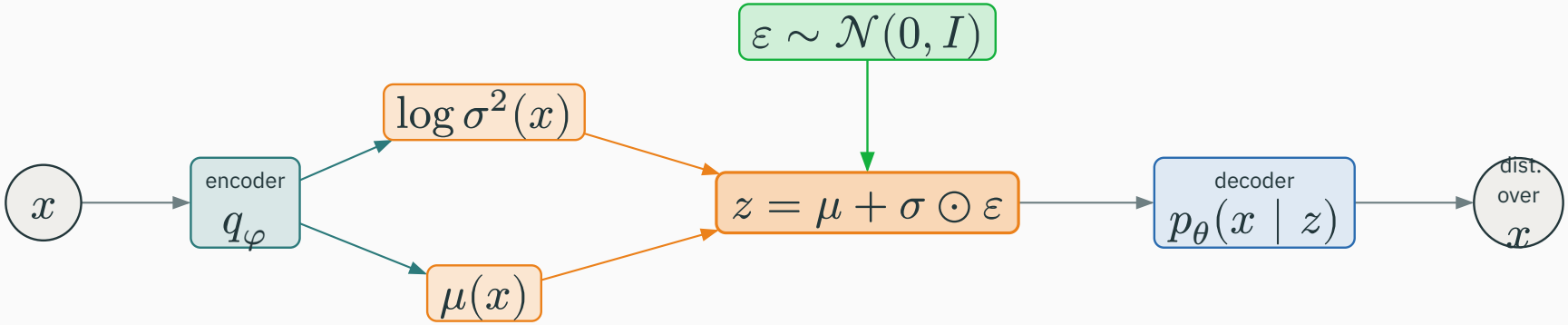

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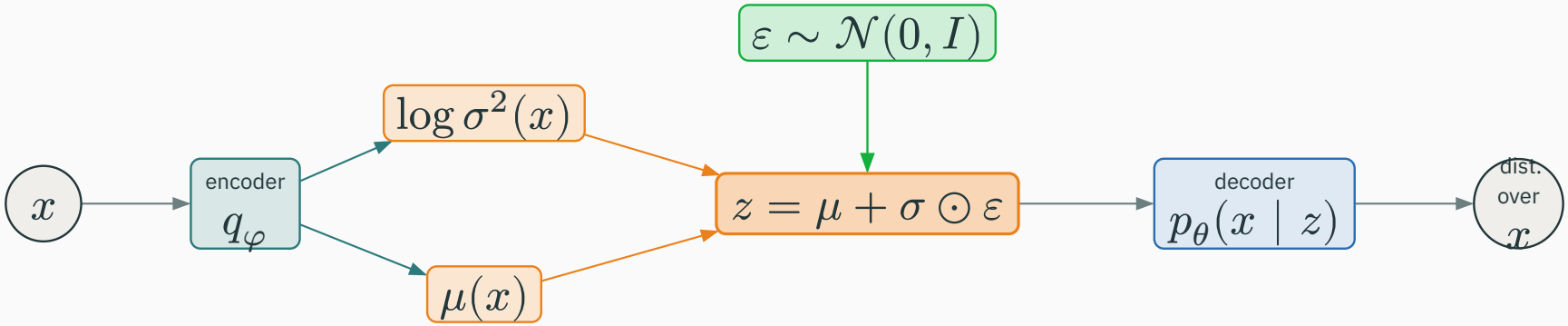

$x \to$ encoder $\to (\mu, \log \sigma^2) \to$ sample $z \to$ decoder $\to$ distribution over x

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two terms, two jobs — this is the object we actually optimize

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Understand what each term **does** before any algebra: the first term **explains the data**, the second **keeps latent codes near the prior** so we can sample them later.

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without it, the encoder would scatter codes into islands again

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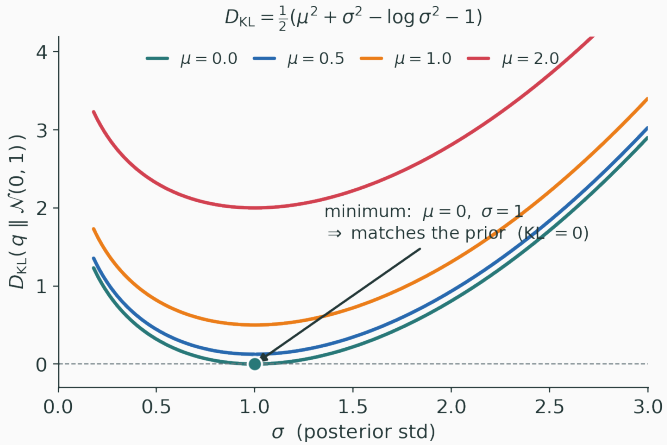

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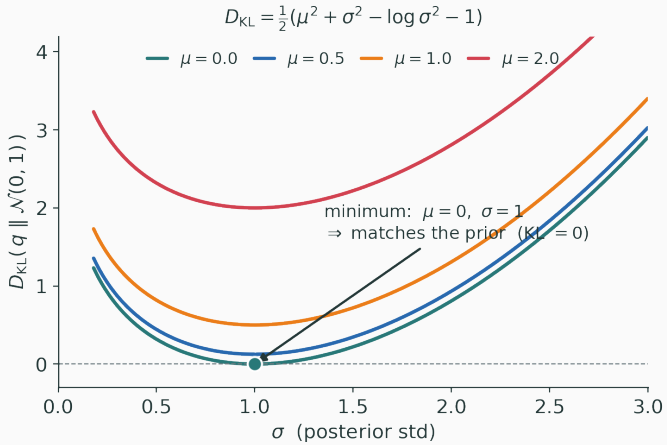

minimized at $\mu = 0, \sigma = 1$ (KL $= 0$) — the KL pushes each code toward the prior

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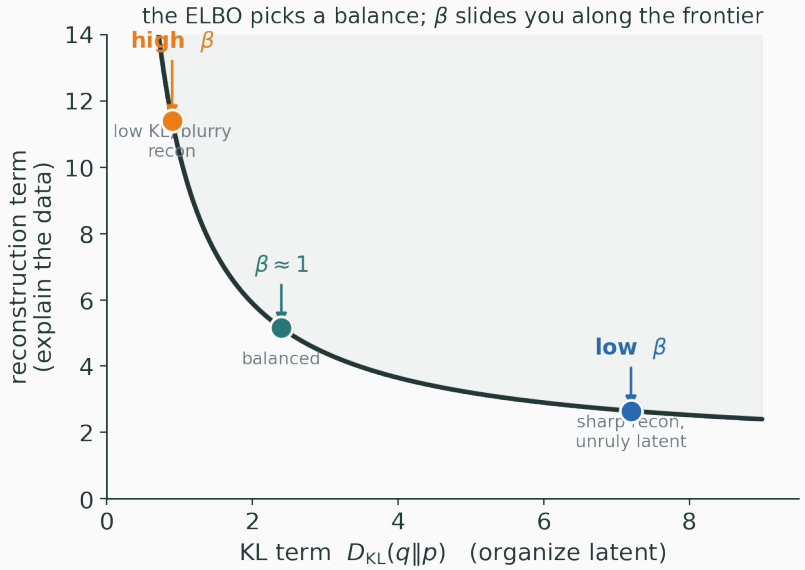

Backpropagation through sampling

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we need the randomness **without** breaking the chain rule

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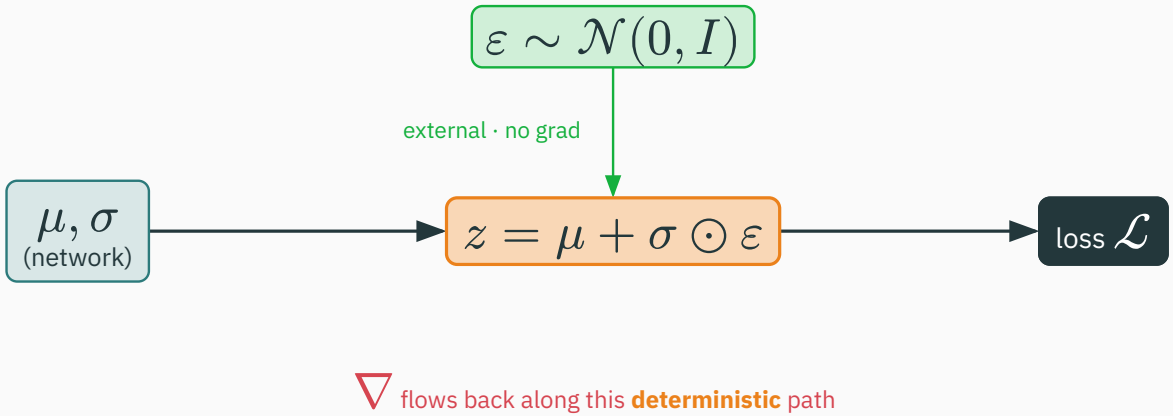

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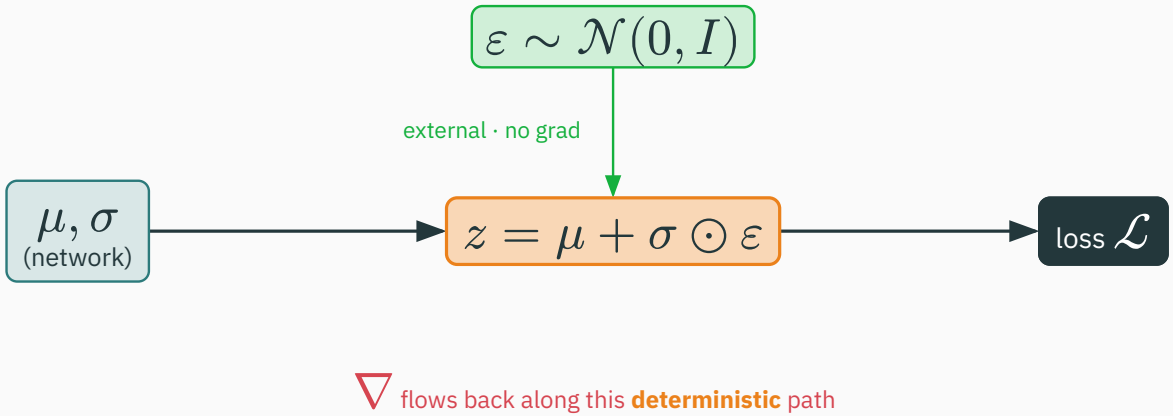

ε is sampled **outside** the network; the map $\mu, \sigma \longrightarrow z$ is a smooth, differentiable function

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grow $\sigma \longrightarrow$ the same ε moves z **further** from μ: more spread, a larger KL

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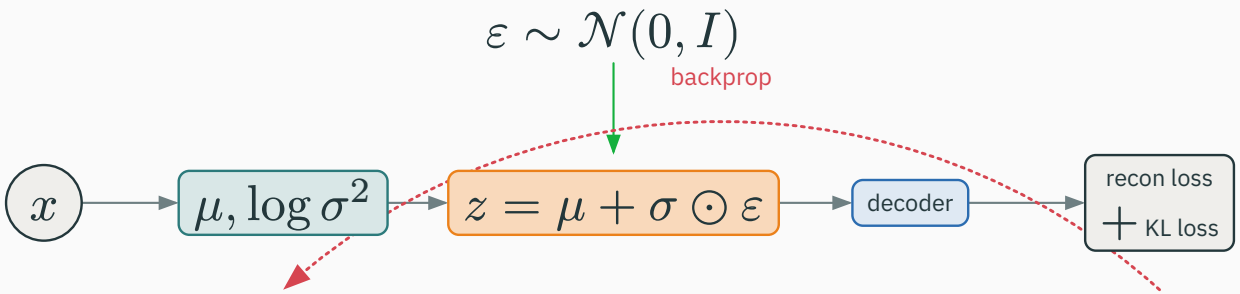

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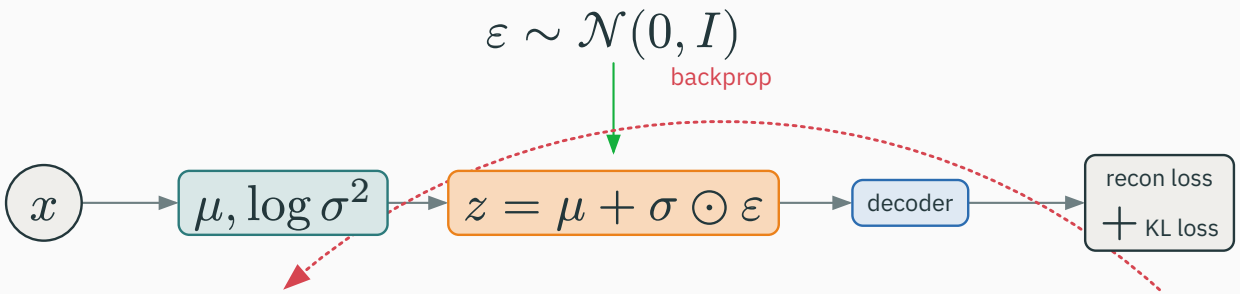
V VISUAL

Every step: encode, reparameterize, decode, score both terms, and backprop.



data **enters** the loop; gradients flow through z into the encoder **and** the decoder

The generation loop

V VISUAL

Generation throws away the encoder — just sample the prior and decode:

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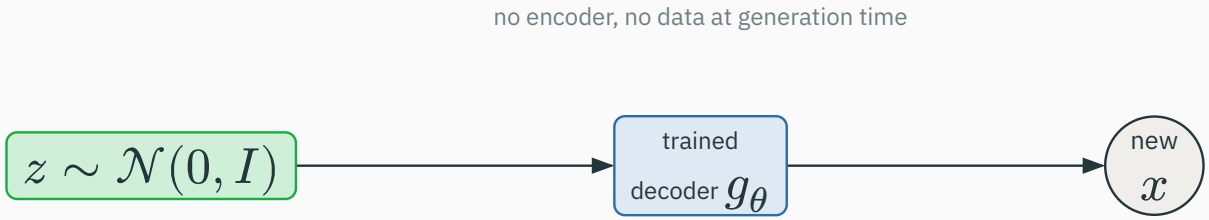

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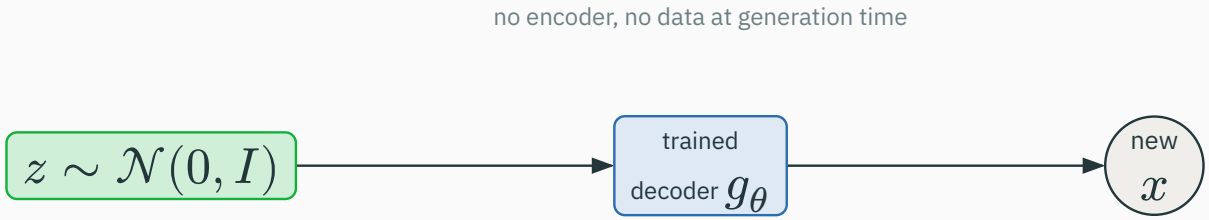

training begins with data; **generation** begins with $z \sim \mathcal{N}(0, I)$ — nothing else

Interpolation as a diagnostic

V VISUAL

Walk a straight line between two codes and decode along the path:

Interpolation as a diagnostic

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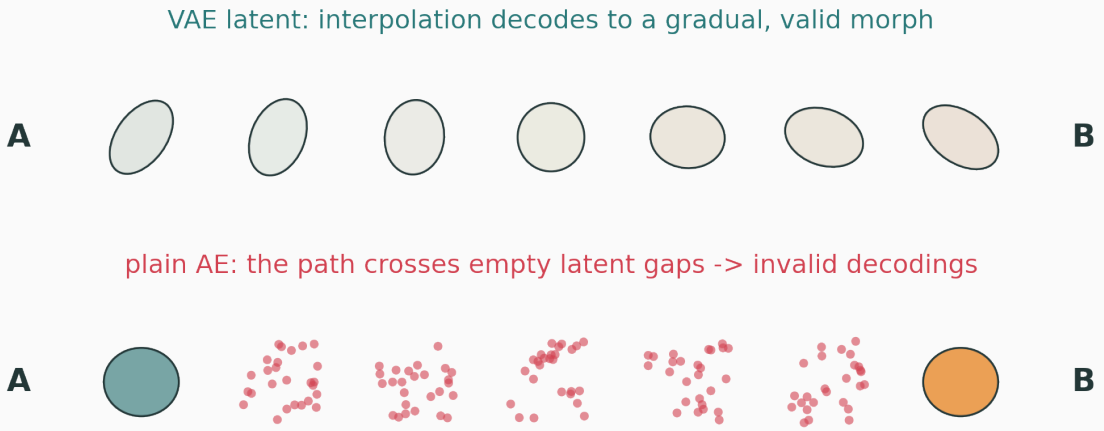

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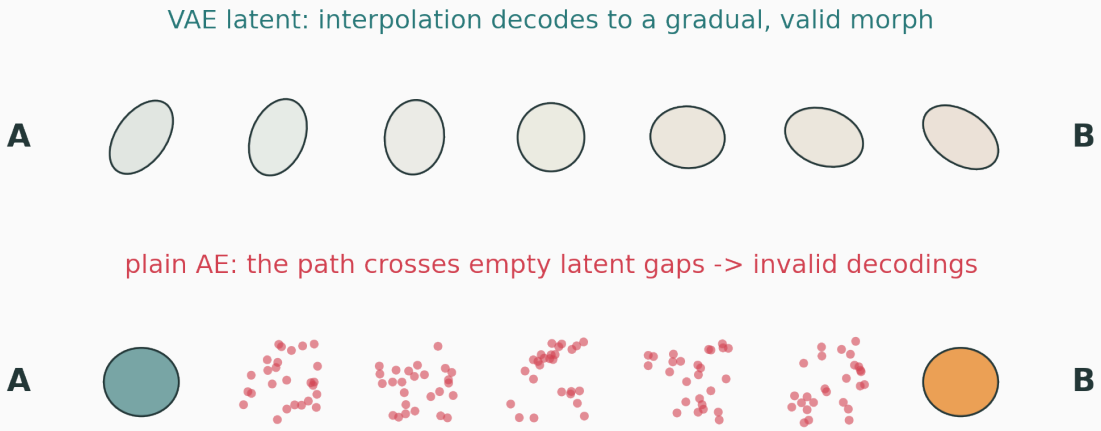

a good latent space yields **gradual semantic change**; a bad one crosses gaps into invalid images

Conditional VAE

D DERIVATION

Feed a **condition** c (a class label, a text embedding) into both networks:

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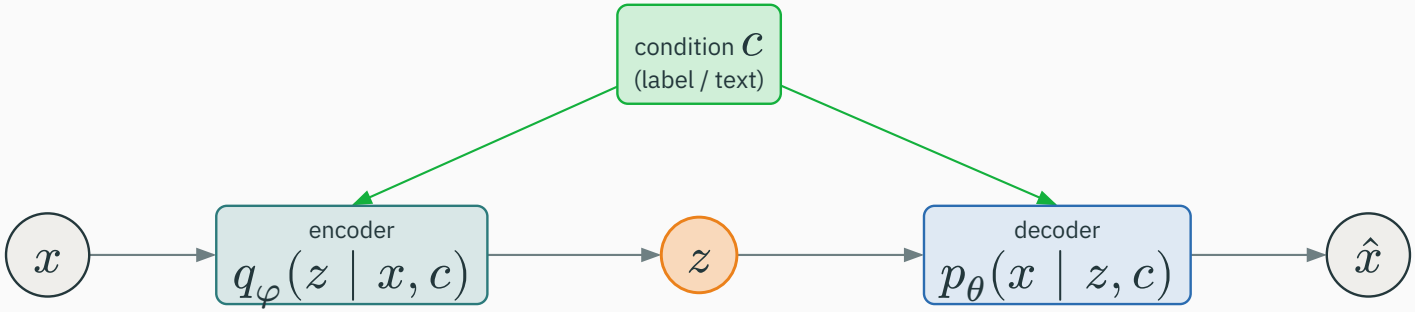

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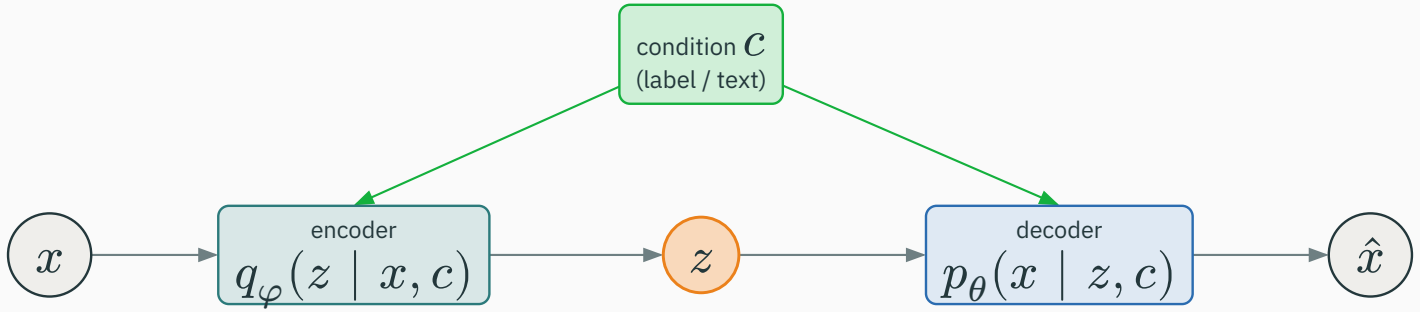

at generation time: **choose** c, sample $z \sim p(z)$, decode — c steers **what** is generated

The β-VAE trade-off

D DERIVATION

Put a **weight** β on the KL term and dial the balance explicitly:

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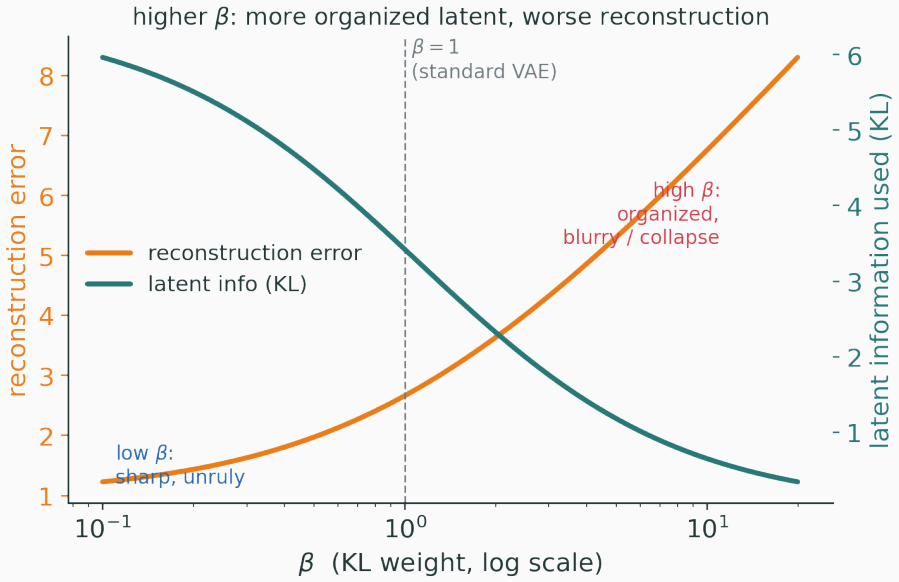

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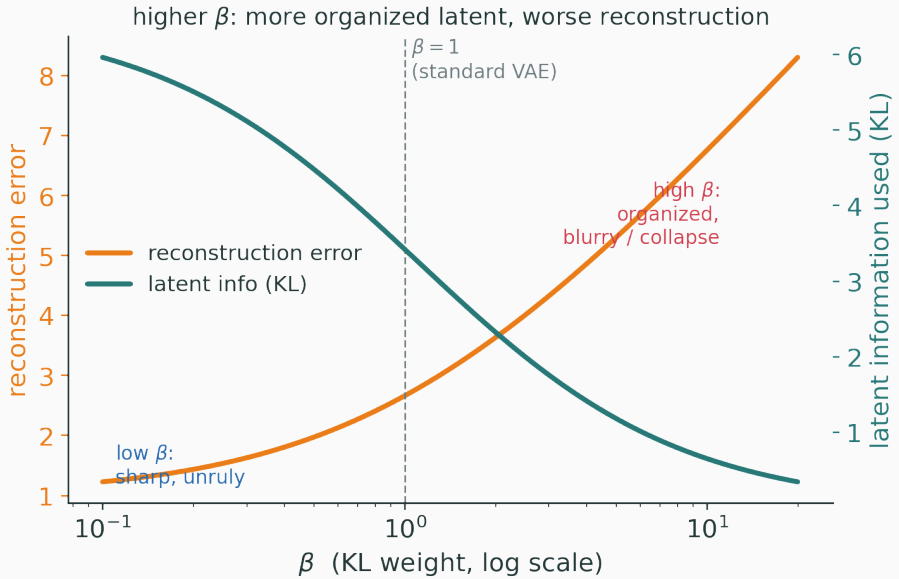

higher $\beta \longrightarrow$ more organized latent factors, but **softer / worse** reconstructions

Honest limitations and the bridge onward

Blurry samples: why?

D DERIVATION

A recurring complaint: VAE samples look **soft**. The cause is the per-pixel likelihood.

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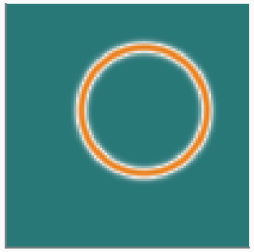
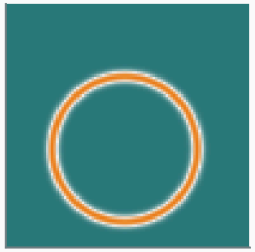
each sample is a plausible sharp output; a per-pixel likelihood rewards their average

sharp sample 1

sharp sample 2



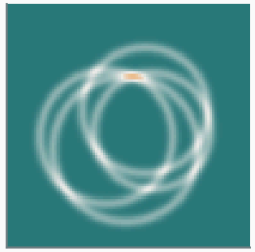
sharp sample 3



+ ⋯

average

pixelwise mean = blurry



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several sharp outputs are all plausible; a per-pixel Gaussian rewards their **average**, which is blurry

Posterior collapse

D DERIVATION

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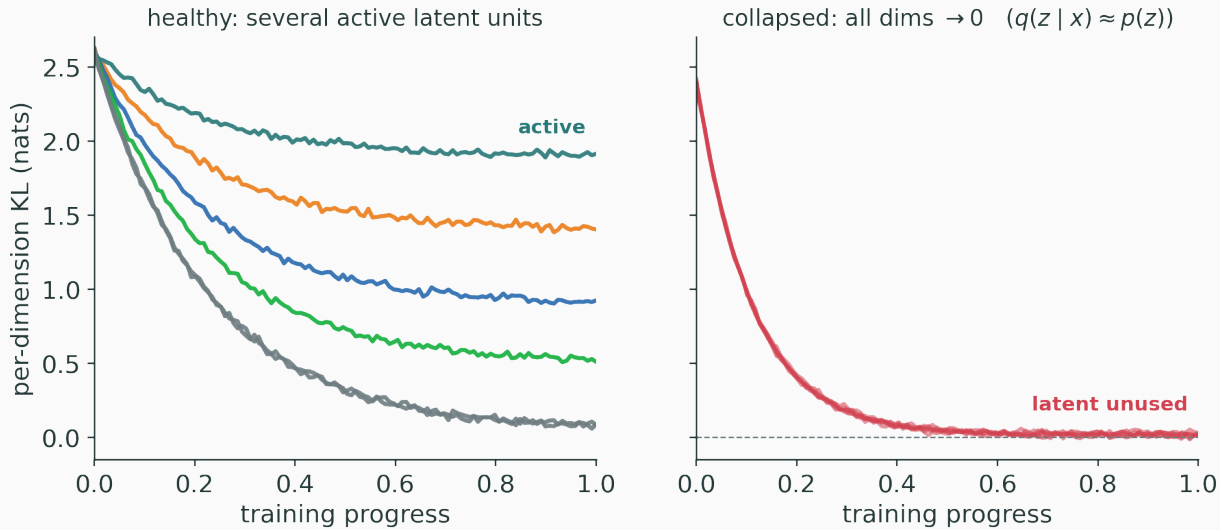

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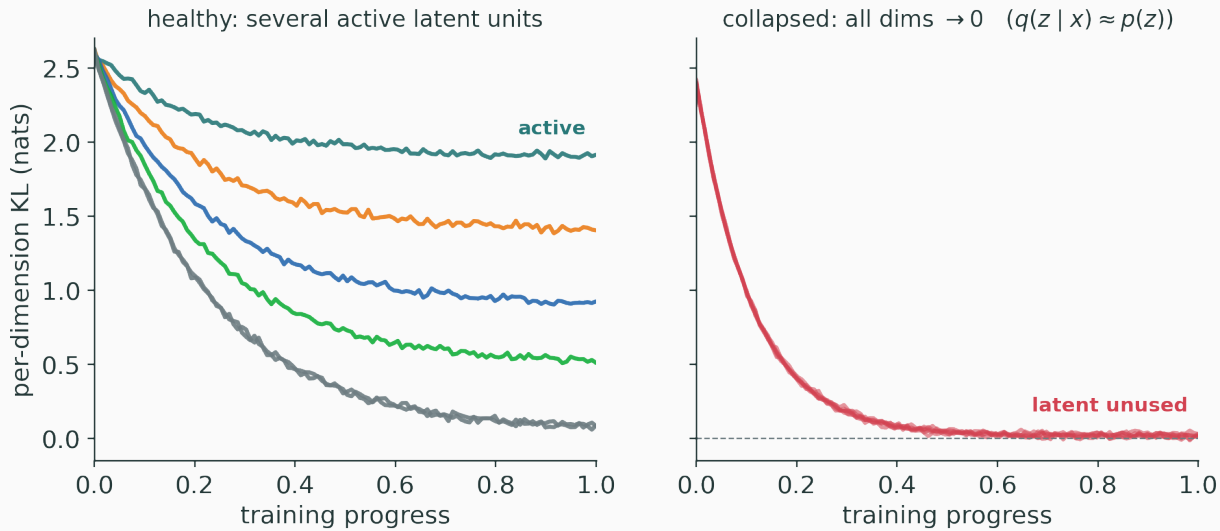

Two routes to the same failure: an over-powerful **autoregressive** decoder can predict x from its own context alone, and — more often for a plain feed-forward

Posterior collapse

D DERIVATION

decoder like ours — the KL term simply dominates. Either way $q_\varphi(z \mid x) \to p(z)$ for every x and the latent goes **unused** — not magically regularized.

VAE versus deterministic autoencoder

V VISUAL

	Autoencoder	VAE
code	one point	a distribution
sampling	unreliable (islands)	defined by the prior
objective	reconstruction	likelihood lower bound
typical output	useful features	smooth, often softer samples

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the KL term is the single change that turns a point-code into a **sampleable** latent space

VAE versus GAN — a preview

D DERIVATION

Both use a latent z, but their **learning signals** differ radically:

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VAEs optimize a bound on $p(x)$; GANs play a game — sharper samples, no density

Connection to diffusion

V VISUAL

Modern **latent diffusion** leans directly on the autoencoder:

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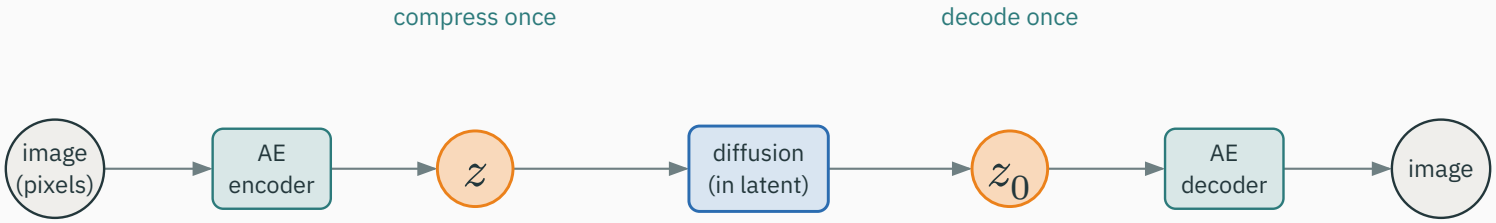

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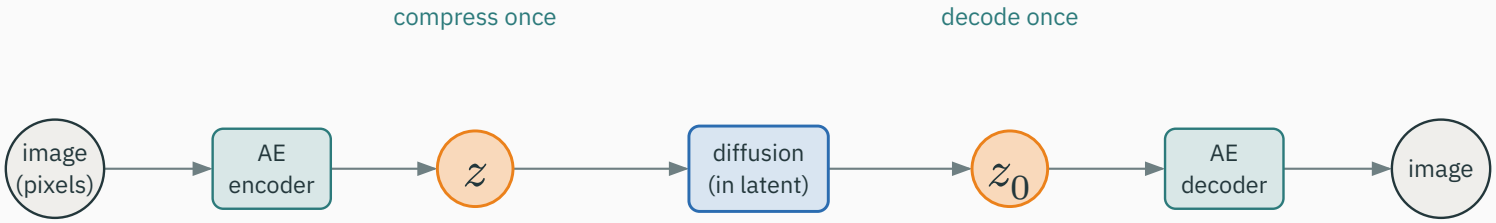

compress pixels to a compact latent, run diffusion **there**, decode once — the AE is an active component

Exit ticket

Q QUESTION

On a small VAE diagram, label the three steps and predict a failure:

Exit ticket

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Label — (1) the **reconstruction** step, (2) the **KL** step, (3) the **sampling** step.

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no KL ⟶ the codes drift back into islands ⟶ prior samples decode to nonsense again

Closing line

Q QUESTION

VAEs make latent spaces sampleable by regularizing them

Closing line

Q QUESTION

VAEs make latent spaces sampleable by regularizing them

Next: **GANs** trade the explicit likelihood for an **adversarial game**

— giving up density estimation to chase **sharper** samples

Deeper: latent-variable models

Autoencoders are nonlinear compression

D DERIVATION

PCA finds the best **linear** low-dimensional reconstruction:

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An autoencoder replaces the two linear maps with **neural networks**, so it can bend around **nonlinear** manifolds.

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an intuition link, **not** a claim that every autoencoder is PCA — the nonlinearity is the point

Shape bookkeeping

D DERIVATION

Track the dimensions as data flows through the encoder:

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A concrete example — a 32×32 grayscale image compressed to a 16-dimensional code:

$$x \in \mathbb{R}^{1{,}024} \quad \longrightarrow \quad z \in \mathbb{R}^{16} \qquad (\text{a } 64\times \text{ squeeze})$$

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what must the 16 numbers preserve so the decoder can rebuild all 1,024 pixels?

Undercomplete and overcomplete codes

V VISUAL

Code dimension	Pressure on the model
$d < \dim(x)$ (undercomplete)	must compress salient structure
$d \geq \dim(x)$ (overcomplete)	may learn an identity shortcut
overcomplete + regularization	can still learn sparse / robust features

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the bottleneck can be **dimensional**, **stochastic**, **architectural**, or **regularization**-based

Reconstruction likelihood, not just MSE

D DERIVATION

Grounding the familiar loss in probability. If the decoder is Gaussian:

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then the negative log-likelihood is **proportional to squared error**:

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MSE is a **modeling choice** (a Gaussian decoder), not a law of nature

Data type determines the decoder output

V VISUAL

Data	Decoder distribution	Reconstruction term
normalized continuous pixels	Gaussian	squared error
binary image	Bernoulli	binary cross-entropy
tokens	categorical	cross-entropy

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the decoder predicts **distribution parameters** — match the distribution to the data type

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V VISUAL

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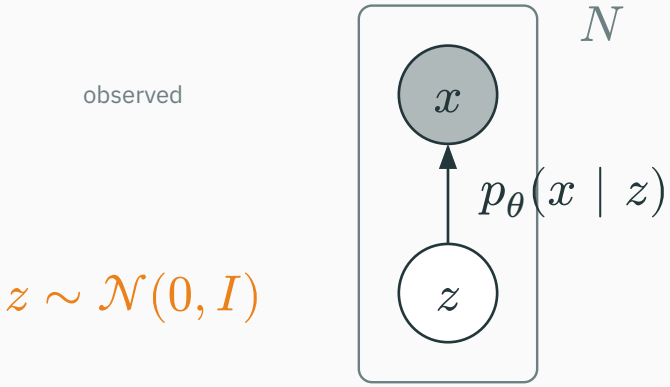

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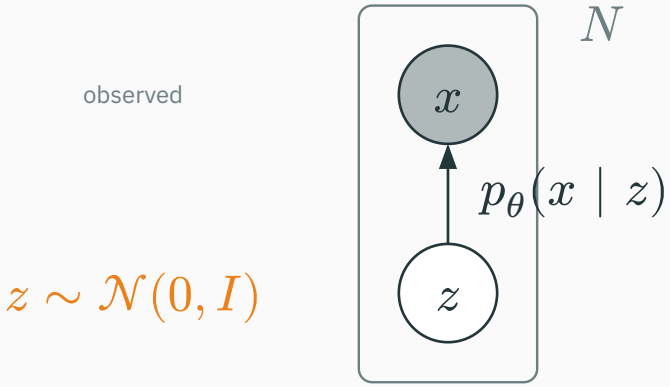

training **inverts** it: infer the hidden cause z that plausibly generated each observed x

Marginal likelihood is an integral

D DERIVATION

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For a nonlinear neural decoder and high-dimensional z, this integral has **no closed form** and cannot be evaluated exactly — hence the lower bound.

Variational inference: a tractable stand-in

D DERIVATION

Choose a tractable family $q_\varphi(z \mid x)$ to **imitate** the true posterior $p_\theta(z \mid x)$:

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classical VI optimizes per-datapoint; the VAE trains one network to predict q for all of them

The exact decomposition

D DERIVATION

The identity behind the bound's **name**:

The exact decomposition

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$$\log p_\theta(x) = \mathrm{ELBO}(x) + D_{\mathrm{KL}}\Big(q_\varphi(z \mid x) \,\|\, p_\theta(z \mid x)\Big)$$

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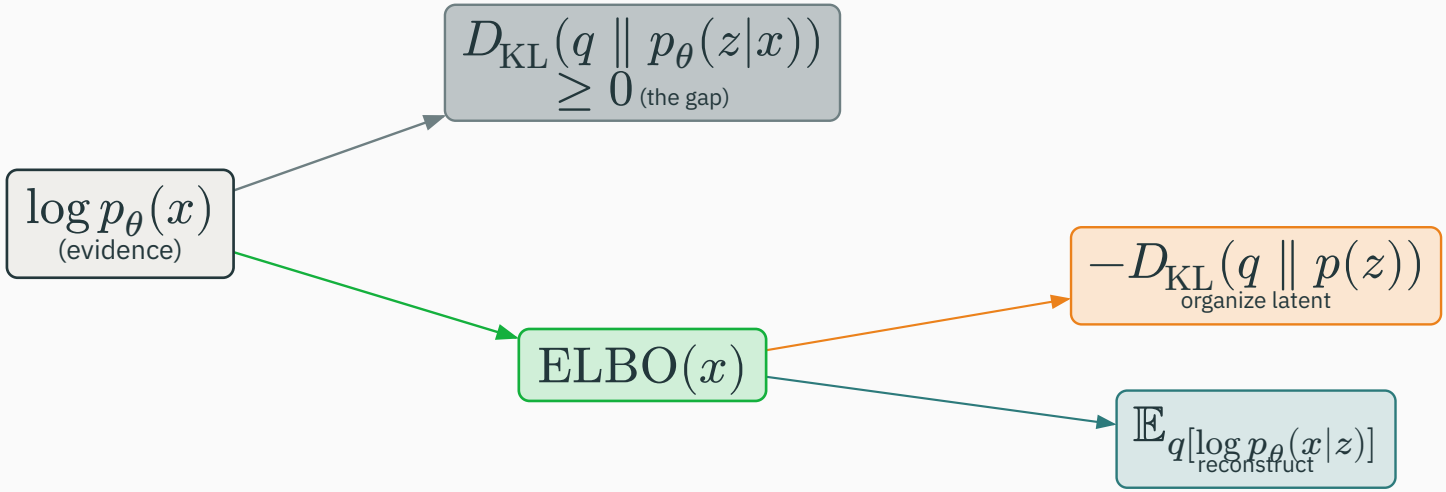

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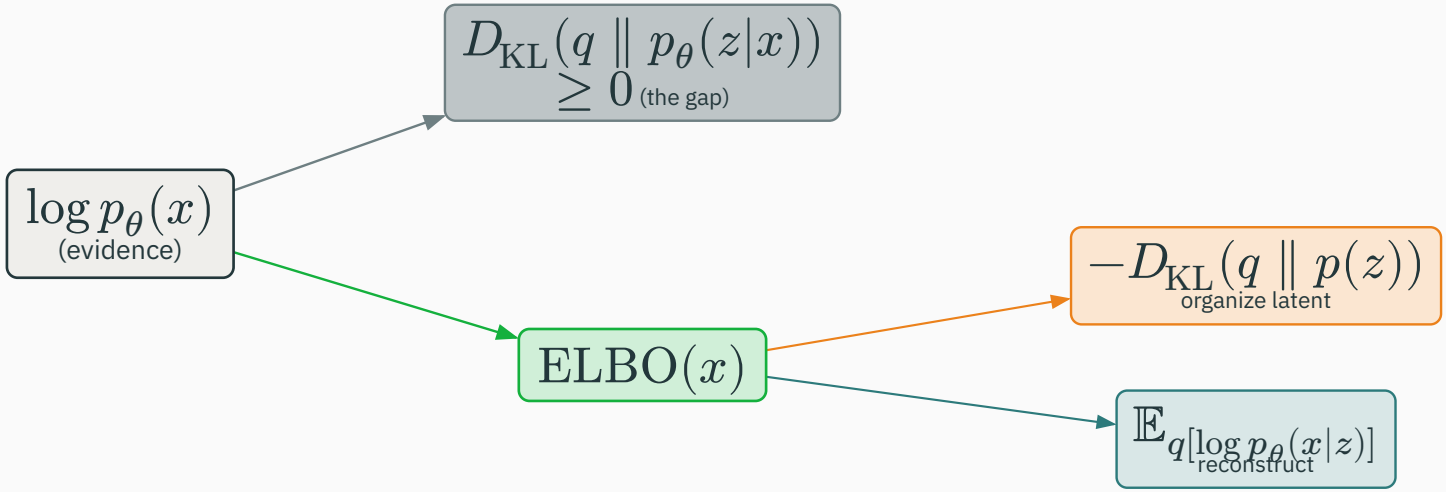

the KL gap is ≥ 0, so ELBO $\leq \log p_\theta(x)$ — a genuine **lower bound**

Expand the ELBO

D DERIVATION

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substitute the joint, then split the logarithm — the next slide collects the pieces

Rearrange into the two familiar terms

D DERIVATION

Grouping the split logarithm gives back the working objective:

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same two terms as Part II — now **derived**, not asserted

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D DERIVATION

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one is a **term you optimize**; the other is **how loose the bound is**

Training loss signs

D DERIVATION

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in code the first term is literally the MSE or BCE; the second is the closed-form Gaussian KL

Monte Carlo estimate

D DERIVATION

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The expectation over $q_\varphi(z \mid x)$ is approximated by **sampling** latent vectors:

$$\mathbb{E}_{q[\cdot]} \approx \frac{1}{K} \sum_{k=1}^{K} [\cdot]_{z^{(k)}}, \qquad z^{(k)} = \mu + \sigma \odot \varepsilon^{(k)}$$

one reparameterized sample per input ($K = 1$) is usually enough for the standard objective

Why output log-variance?

D DERIVATION

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$$\text{e.g. } \ell = -1.386 \to \sigma = \exp(-0.693) = 0.5$$

Reparameterization computation graph

V VISUAL

The full graph with the log-variance parameterization:

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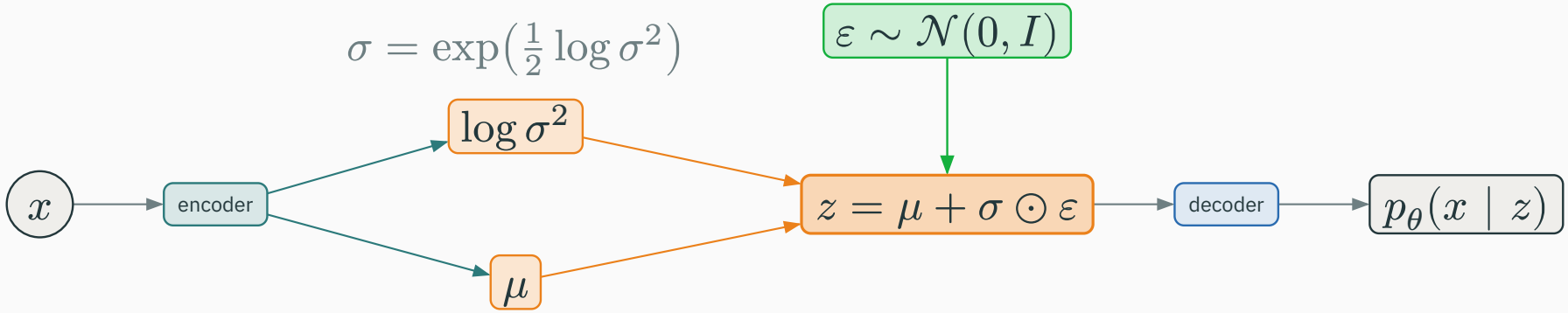

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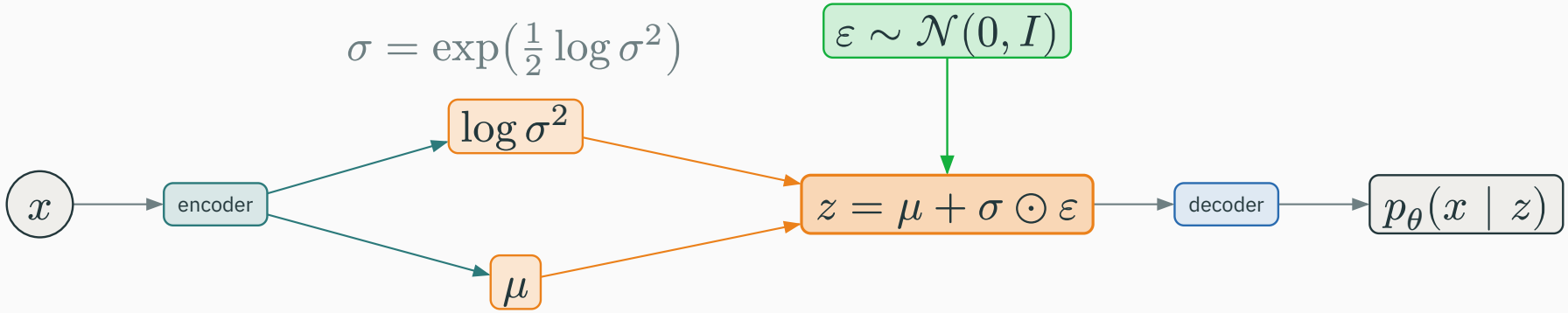

ε is drawn independently; gradients flow through the **deterministic** transform $\mu, \sigma \to z$

Worked: one training step's loss

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Take the same encoded point, $\mu = [0.8, -0.7]$, $\sigma = [0.5, 0.2]$, with $\sigma_x^2 = 1$ and $\|x - \hat{x}\|^2 = 3.0$:

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both terms are in **nats**; the optimizer trades one against the other

Sampling versus reconstruction

V VISUAL

Operation	Latent source	Question answered
reconstruction	$z \sim q(z \mid x)$	can the model explain this input?
unconditional sample	$z \sim p(z)$	what does the model generate?
interpolation	path between two codes	is the latent geometry smooth?

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where z comes from changes the meaning of the decoder's output

A KL trade-off numeric intuition

D DERIVATION

Two candidate encoders for the **same** data:

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Encoder	Reconstruction	KL
highly individualized codes	1	100
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totals 101 vs 11: neither is “obviously best” — the **objective** picks the balance, not reconstruction alone

Visualize a 2-D latent space

V VISUAL

Encode a dataset into (z_1, z_2) and decode a **grid** of latent points:

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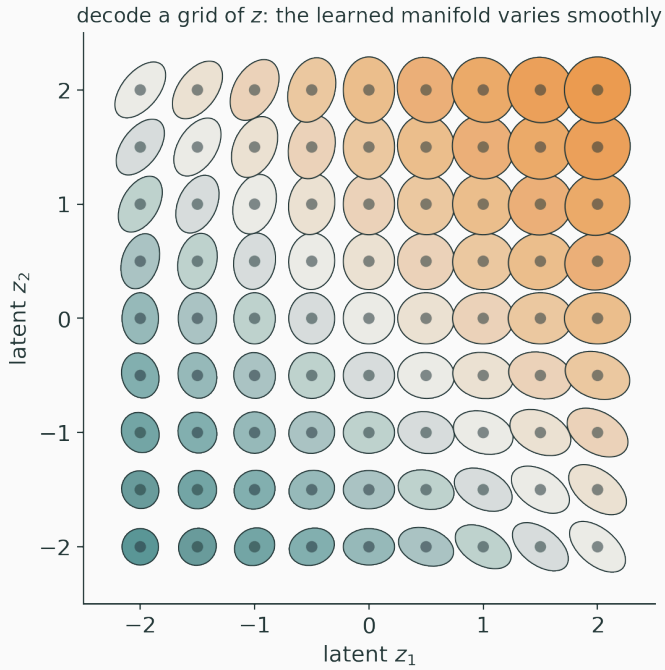

Visualize a 2-D latent space

V VISUAL

Encode a dataset into (z_1, z_2) and decode a **grid** of latent points:

a good map shows **clusters**, **smooth directions**, **empty regions**, and where **prior samples** land

Latent traversal

D DERIVATION

Hold every coordinate fixed and move a **single** latent dimension:

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Interpretable changes along z_j are **interesting evidence** about the learned representation — but **not proof** that the factors are perfectly disentangled.

Conditional VAE, expanded

D DERIVATION

For a label or text condition c, condition **both** networks:

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Still diverse through z — the style, stroke, and variation **within** that attribute.

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sample: choose c, draw $z \sim p(z)$, decode $p_\theta(x \mid z, c)$

β-VAE and disentanglement claims

D DERIVATION

$$\mathcal{L} = -\mathbb{E}_{q[\log p_\theta(x \mid z)]} + \beta\, D_{\mathrm{KL}}(q \parallel p)$$

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It **can** encourage more separated factors on curated data — but "disentangled" is **not** a guaranteed, universal outcome, and reconstruction suffers.

Posterior collapse, diagnose it

D DERIVATION

The symptom to watch for:

Posterior collapse, diagnose it

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$q_\varphi(z \mid x) \approx p(z)$ for **every** x

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A powerful decoder predicts x well from context alone, so the KL term drives every posterior to the prior. The latent has become **unused** — inspect per-dimension KL, not just reconstruction.

Posterior collapse, high-level responses

D DERIVATION

Design choices, not a new module:

Posterior collapse, high-level responses

D DERIVATION

Design choices, not a new module:

- **weaken or schedule** the KL pressure early in training (KL annealing);
- **constrain** the decoder's capacity so it cannot ignore z;
- **force** more information through the latent (e.g. a free-bits floor);
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the fixes all **rebalance** how much the model is allowed to rely on the decoder alone

VQ-style discrete latents

D DERIVATION

Some autoencoders encode into a **finite codebook** instead of a continuous Gaussian:

VQ-style discrete latents

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INTERACTIVE

↗ nipunbatra.github.io/interactive-articles/vq-codebook

A codebook widget: snap continuous encodings to their nearest discrete code vectors.

Why VAEs matter to modern diffusion

V VISUAL

Latent diffusion uses the autoencoder to change **what space** the generator works in:

Why VAEs matter to modern diffusion

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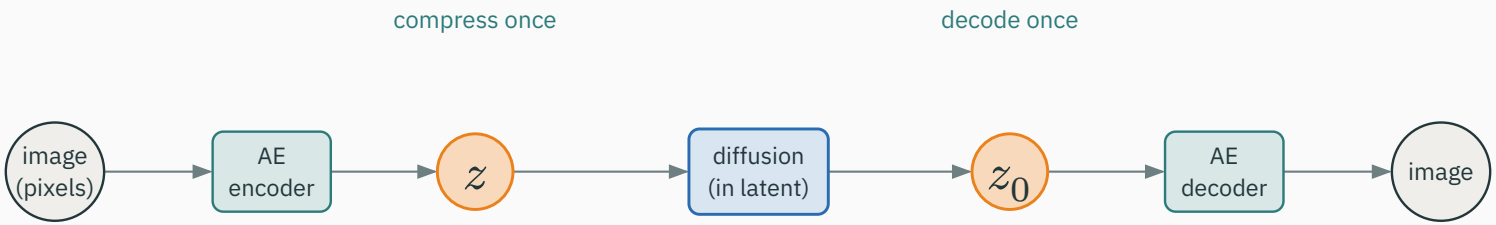

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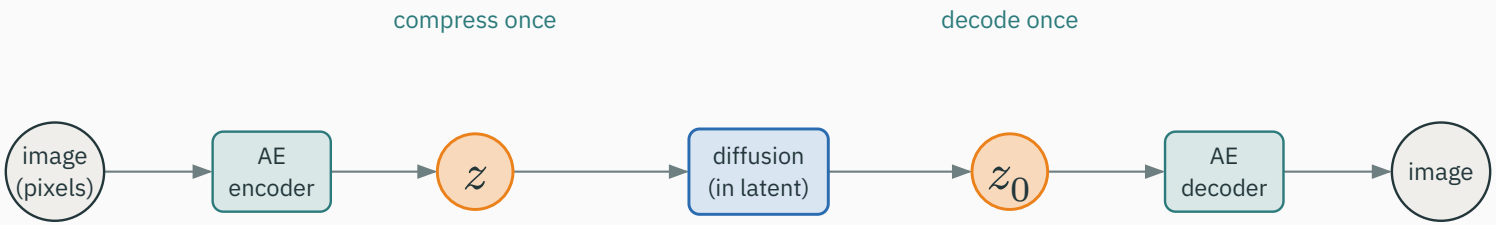

the VAE is not historical background — it is an **active systems component** of latent diffusion

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Q QUESTION

No single number suffices — use several lenses:

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- **sample quality and diversity** (are outputs realistic **and** varied?);
- **reconstruction / likelihood** metrics where appropriate;
- **downstream utility** of the learned features;
- **human evaluation** with a transparent protocol;
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a model can score well on one lens and fail badly on another

Common misconceptions

V VISUAL

Misconception	Correction
“the encoder outputs one latent point”	it outputs a **distribution’s parameters**
“KL makes samples better directly”	it **organizes codes** to match the sampling prior
“blurry samples prove VAEs are wrong”	they reflect likelihood / output trade-offs
“interpolation proves understanding”	it is a **diagnostic**, not proof

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4. What is **posterior collapse**, and how would you **detect** it?

Next lecture

Q QUESTION

VAE: explain data through a latent **likelihood**

↓

GAN: learn realism through an adversarial **critic**

Next lecture

Q QUESTION

VAE: explain data through a latent **likelihood**

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GAN: learn realism through an adversarial **critic**

the next model **removes** the explicit reconstruction target and introduces a **game**

A VAE turns a code into a **sampleable latent variable**: reconstruction $+$ a KL-regularized prior $=$ a generative model.

Next: **Generative Adversarial Networks** $\rightarrow$ diffusion $\rightarrow$ modern generative models.