

Making Deep Networks Trainable

Activations → initialization → normalization → residual connections

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ES 667 · Deep Learning · IIT Gandhinagar

Backprop found the gradients; depth may erase them

L4 · BACKPROP

computes $\nabla_{\theta} \mathcal{L}$

L5 · OPTIMIZATION

chooses the parameter
update

L6 · TRAINABILITY

asks whether signals and
gradients survive depth

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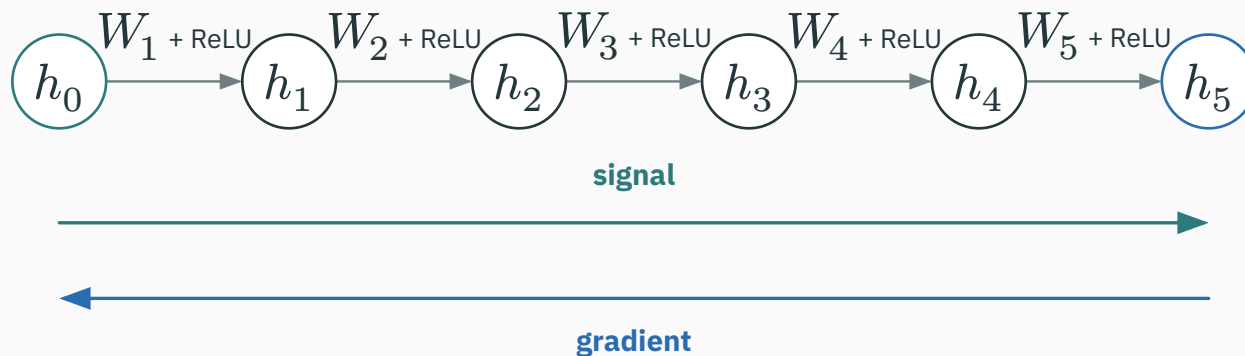
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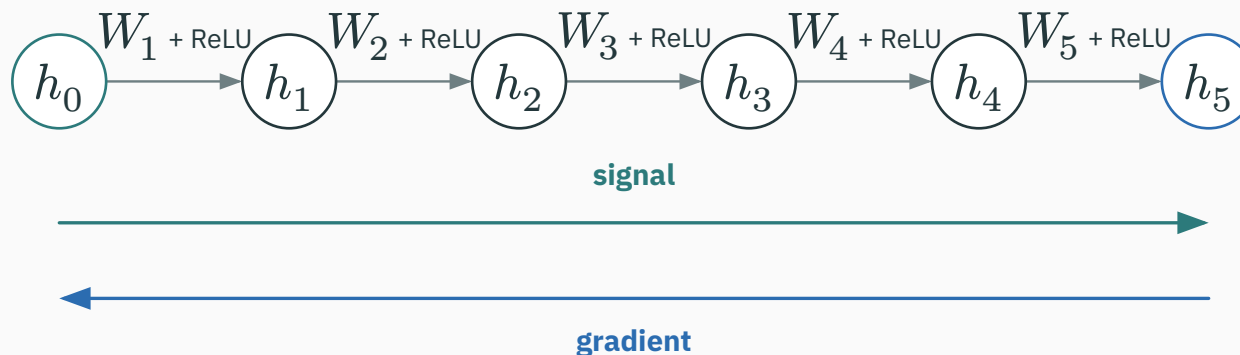
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Today the computation rule stays fixed; we redesign how information travels through it.

Commit: which start survives five ReLU layers?

QUESTION

ANCHOR · five width-4 layers · ReLU after every affine map · $q_0 = 1 \cdot g_5 = 1$

Which zero-mean weight distribution should keep both end-to-end RMS gauges closest to 1 at initialization?

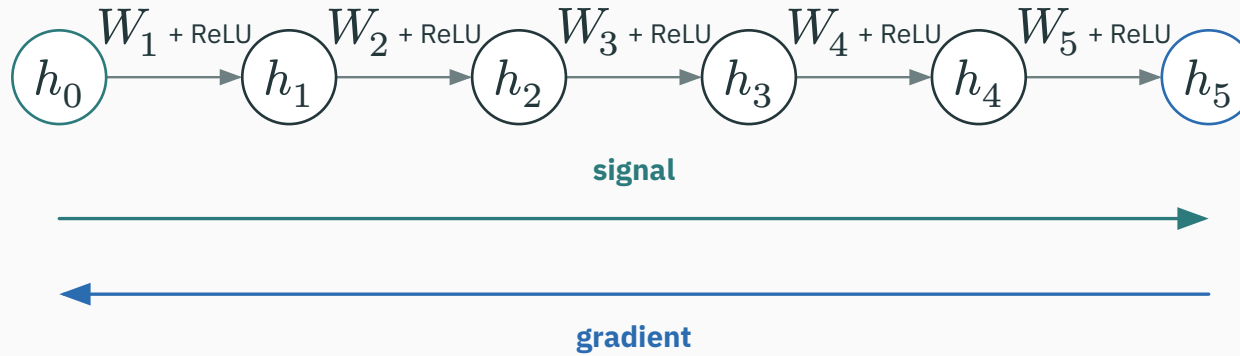
A. Xavier normal with $\text{Var}(w) = \frac{1}{4}$

B. Xavier uniform with the same variance $\frac{1}{4}$

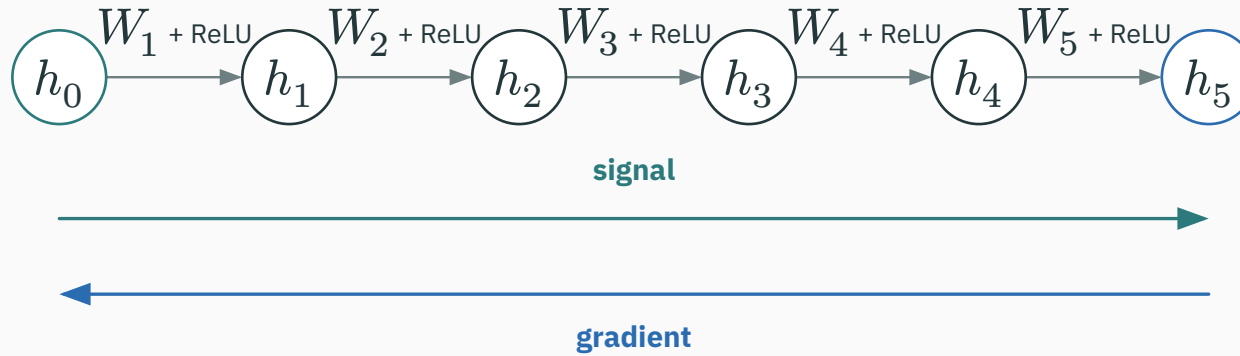
C. He normal with $\text{Var}(w) = \frac{1}{2}$

D. All weights equal to zero

Keep your bet; every section updates one ledger



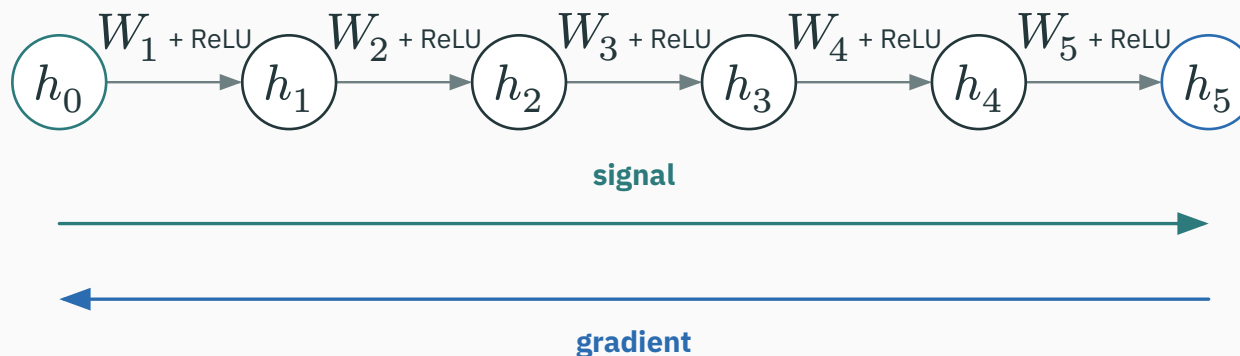
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We will calculate the same chain, then add one control at a time:



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Do not change your answer yet. We return to it after the activation gate determines the matching initialization.

Outline: keep signals usable through depth



**Predict the drift → measure both RMS gauges → change the earliest plausible cause
→ measure again.**

— forward signal — backward gradient — chosen gain / reset — preserved / identity — failure

For width d_ℓ , average over coordinates—not a raw vector norm:

$$q_\ell := \frac{1}{d_\ell} \sum_i (h_{\ell,i})^2, \quad r_\ell := \sqrt{q_\ell}$$

$$g_\ell := \frac{1}{d_\ell} \sum_i (\bar{h}_{\ell,i})^2, \quad s_\ell := \sqrt{g_\ell}.$$

One gauge, two directions, no width drift

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r_ℓ is activation RMS; q_ℓ is its squared ledger

BACKWARD

s_ℓ is gradient RMS; g_ℓ is its squared ledger

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per-layer c	c^1	c^5	RMS factor $\sqrt{c^5}$
0.5	0.5	0.03125	0.17678
1	1	1	1
2	2	32	5.65685

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Depth turns a small scale mismatch into a visible vanishing or exploding trace.

The repair order follows causality



activation	sets the local nonlinear gate
initialization	matches weight gain to that gate at $t = 0$
normalization	re-establishes a reference scale while training
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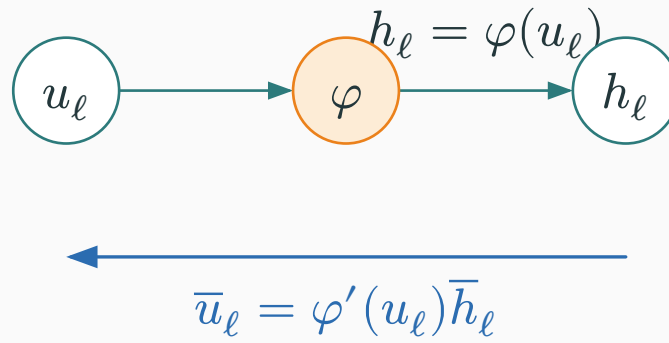


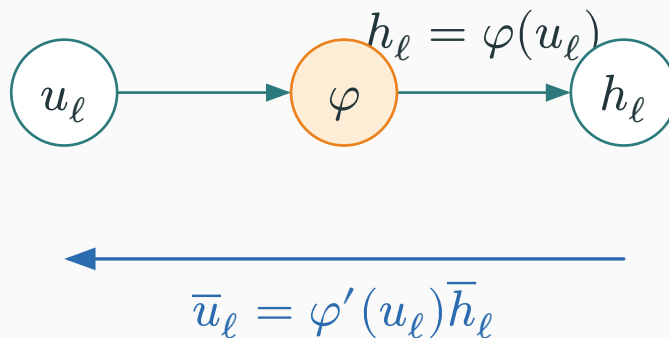
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Activation comes first because its gate determines the scale that initialization must match.

**The activation is a gradient
gate**

Every nonlinearity acts in both passes





The curve shapes the representation; its derivative gates the returning gradient.

Sigmoid becomes nearly flat in either tail

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| Even its best slope is only $\frac{1}{4}$; saturation makes the slope far smaller.

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D DERIVATION

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Only 0.665% of the arriving gradient magnitude crosses this activation.

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ReLU passes the active-side gradient unchanged and blocks the inactive side.

The anchor is half active in expectation

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The gate contributes $\frac{1}{2}$ to both squared-gauge multipliers.

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large negative bias, overly large update,
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count examples active for each unit—
not only the global zero fraction

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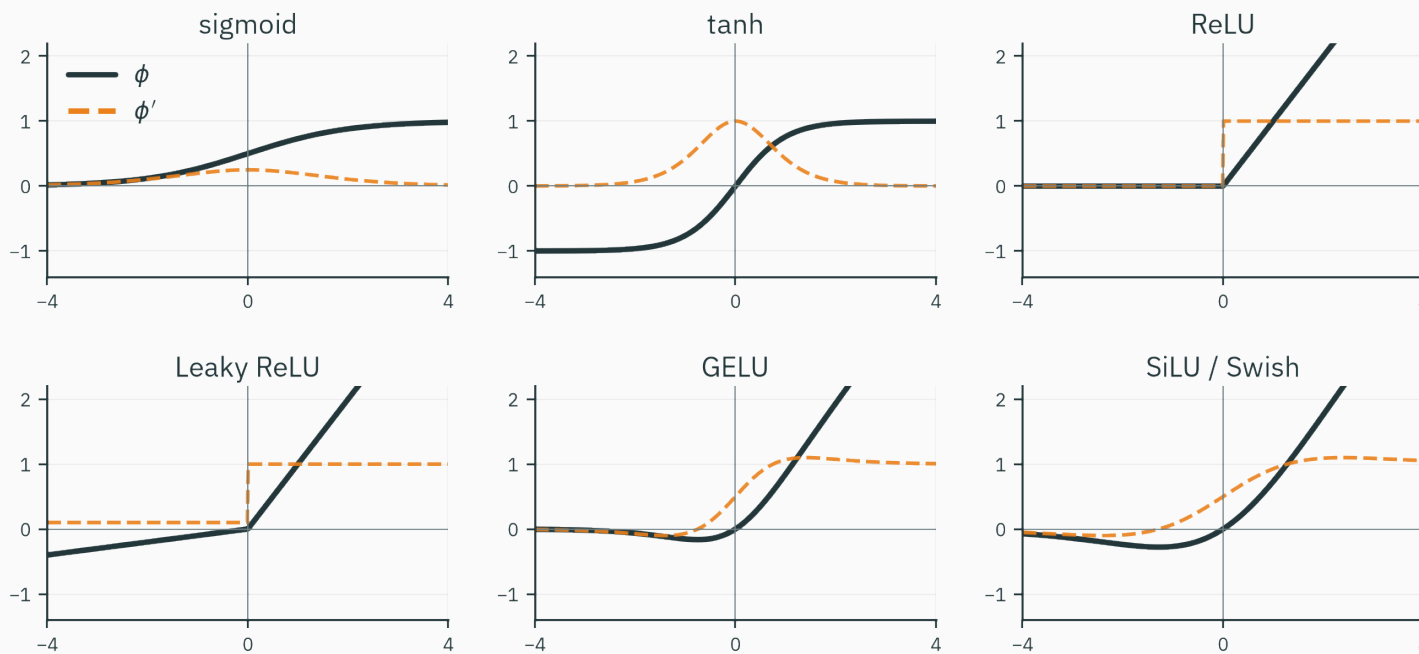
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Changing upstream representations may revive it; “can never recover” is too strong.

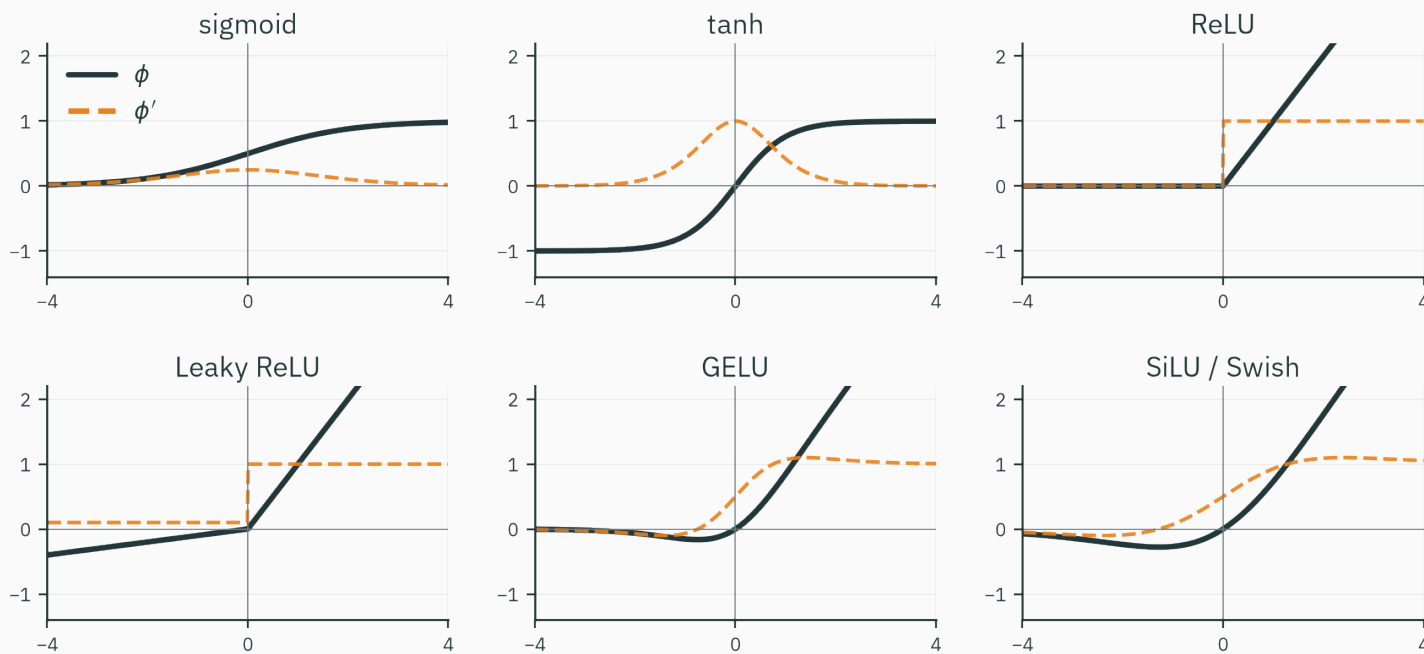
Compare the gate, not only the curve



Solid: $\varphi(u)$. Dashed: $\varphi'(u)$, the returning-gradient gate. GELU uses the documented sigmoid approximation $u\sigma(1.702u)$; the other curves are exact away from their kink.

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Choose the activation for representation; choose initialization to match its gain.

GAUGE LEDGER · SAME FIVE-LAYER ANCHOR

control	evidence / status · squared q, g ; RMS r, s where shown
activation	$c_{\text{gate}} = \frac{1}{2}$ in both squared gauges
initialization	not chosen; weight gain still unknown
normalization	not yet added
residual path	not yet added

Ledger 1 — the gate is now known

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**Initialization must match the
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An affine layer multiplies the squared signal gauge by $n_{\text{in}} \text{Var}(w)$.

The gate tells us which weight variance to choose

ANCHOR · five width-4 layers · ReLU after every affine map · $q_0 = 1 \cdot g_5 = 1$

For width 4 followed by ReLU,

$$c = \underbrace{4 \operatorname{Var}(w)}_{\text{affine gain}} \times \underbrace{\frac{1}{2}}_{\text{ReLU gate}} = 2 \operatorname{Var}(w).$$

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scheme	$\operatorname{Var}(w)$	one-layer c	five-layer c^5
Xavier	$\frac{1}{4}$	0.5	0.03125
He	$\frac{2}{4} = \frac{1}{2}$	1	1

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The same half-active gate that lost energy supplies the missing factor of 2.

Answer: He keeps the anchor gauges near one

A ANSWER

Correct: C — He normal with $\text{Var}(w) = \frac{1}{2}$

Why: For width 4, affine gain $4(\frac{1}{2}) = 2$ and the ReLU gate contributes $\frac{1}{2}$, so the net squared-gauge factor is $c = 1$.

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Xavier normal and Xavier uniform share a variance, so both give $c = 0.5$ here. Their samples are variance-matched alternatives, not identical distributions.

Xavier loses half the squared gauge per ReLU layer

ANCHOR · five width-4 layers · ReLU after every affine map · $q_0 = 1 \cdot g_5 = 1$

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Xavier preserves the affine pre-activation scale, but ReLU then removes half the second moment.

He preserves the second moment through ReLU

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With $\text{Var}(w) = \frac{2}{n_{\text{in}}} = \frac{1}{2}$,

$$c = 4 \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) = 1.$$

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Under the mean-field assumptions, He preserves activation and gradient RMS through the ReLU stack.

The computed trace stays on one honest scale

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squared gauge	$\ell = 0$	1	2	3	5
Xavier 0.5^ℓ	1	0.5	0.25	0.125	0.03125
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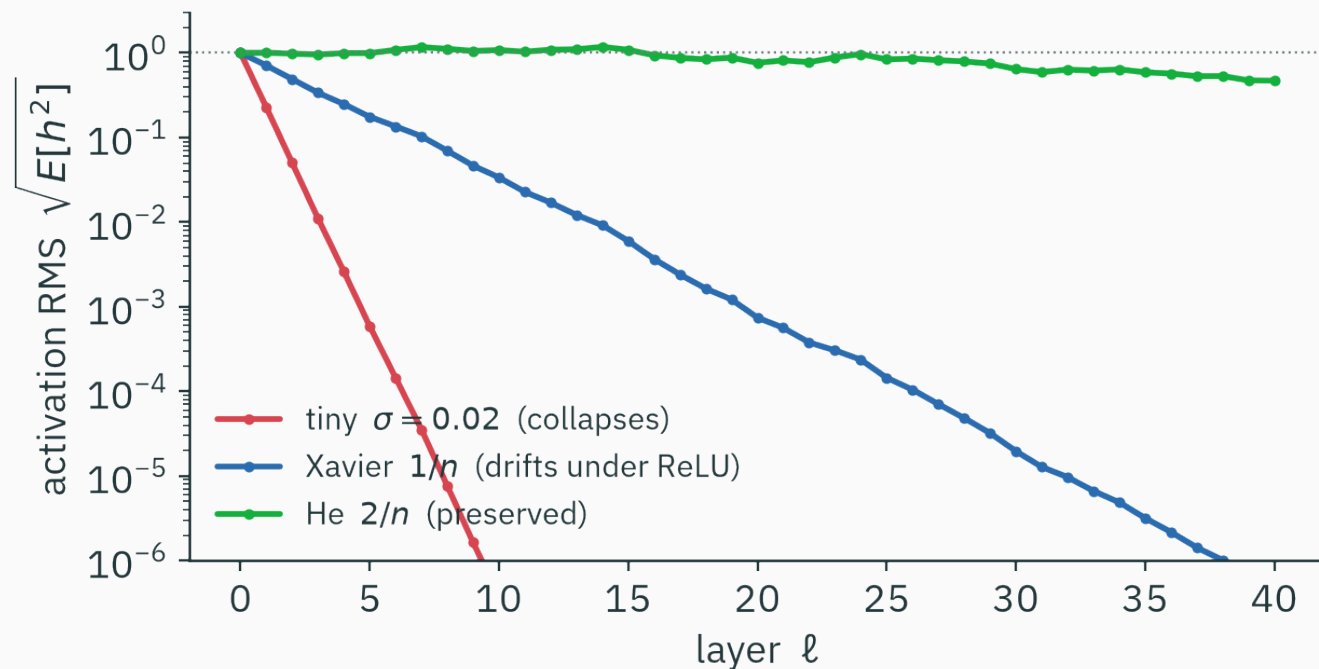
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Both rows share the same exact 0–1 scale; no trajectory falls below or leaves the shown range.

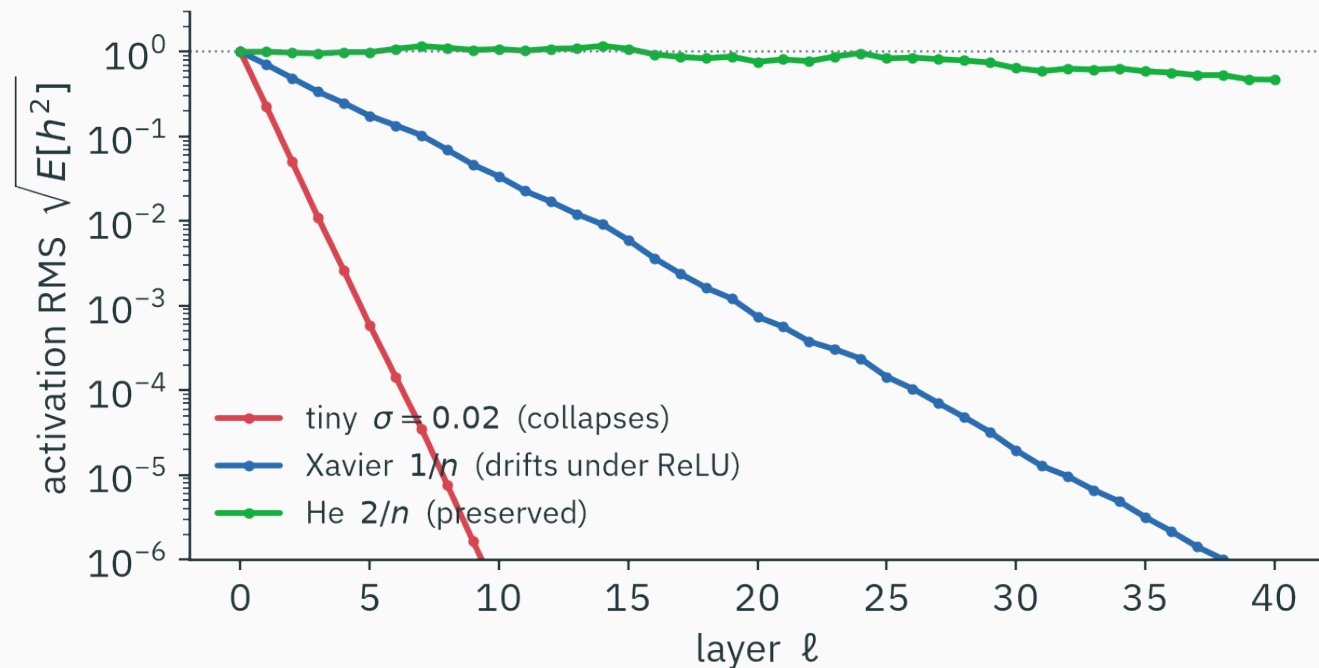
A seeded wider simulation shows the same scale story



Constructed diagnostic, seed 0: width 256, batch 1024, 40 bias-free ReLU layers, Gaussian inputs. Each curve uses newly sampled weights; points are measured activation RMS.

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He keeps the trace order-one in this finite-width draw; Xavier loses roughly one half of the squared gauge per ReLU layer, as predicted.

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“Variance-matched” is the useful equivalence; the distributions themselves are different.

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Stride changes output resolution, not how many input values feed one output. A projection shortcut may use a 1×1 convolution with stride > 1 when it must downsample.

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PROMISE

a sensible RMS scale at $t = 0$

NOT A PROMISE

exact preservation for every layer,
example, or training step

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Training breaks these assumptions: correlations grow, weights move, activation means shift, and data are not idealized.

PROMISE

a sensible RMS scale at $t = 0$

NOT A PROMISE

exact preservation for every layer,
example, or training step

Initialization earns a healthy start; measurement tells us whether it stays healthy.

Ledger 2 — the anchor starts balanced

GAUGE LEDGER · SAME FIVE-LAYER ANCHOR

control	evidence / status · squared q, g ; RMS r, s where shown
activation	ReLU gate contributes $\frac{1}{2}$
initialization	Xavier: $q_5 = g_0 = .03125$; He: $q_5 = g_0 = 1$
normalization	not yet added
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Primary RMS reading: Xavier $r_5 = s_0 = .17678$; He $r_5 = s_0 = 1$.

**Normalization resets a
reference scale**

A normalization layer defines its own measuring group

For values in one chosen group,

$$\mu = \frac{1}{N} \sum_i x_i, \quad \sigma^2 = \frac{1}{N} \sum_i (x_i - \mu)^2$$

$$\hat{x}_i = \frac{x_i - \mu}{\sqrt{\sigma^2 + \varepsilon}}, \quad y_i = \gamma \hat{x}_i + \beta.$$

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$\hat{x}$ has mean near 0 and variance near 1
over that group

LEARN

γ, β restore any useful affine scale and
shift

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The key design question is not “normalize?” but “which entries share one statistic?”

BatchNorm and LayerNorm choose different axes

For $X \in \mathbb{R}^{B \times D}$, rows are examples and columns are features.



BatchNorm: down one feature

reduce over the batch axis B



LayerNorm: across one example

reduce over that example's D features

For one BatchNorm feature across four examples,

$$x = [1, 2, 3, 4], \quad \gamma = 2, \quad \beta = 1, \quad \varepsilon \approx 0.$$

What are the batch mean and population variance used on this slide?

A. $\mu = 2.5, \sigma^2 = 1.25$

B. $\mu = 2.5, \sigma^2 = \frac{5}{3}$

C. $\mu = 10, \sigma^2 = 5$

D. $\mu = 0, \sigma^2 = 1$

Answer: build the whole BatchNorm slice

A ANSWER

$$\mu = \frac{1 + 2 + 3 + 4}{4} = 2.5.$$

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$$\mu = \frac{1 + 2 + 3 + 4}{4} = 2.5.$$

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$$\hat{x} = \frac{x - 2.5}{\sqrt{1.25}} \approx [-1.342, -0.447, 0.447, 1.342].$$

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$$y = 2\hat{x} + 1 \approx [-1.683, 0.106, 1.894, 3.683].$$

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Correct: A — the normalized slice has mean 0, variance 1, and RMS 1 before γ, β .

Learnable scale is allowed to move the gauge

For the normalized slice,

$$\frac{1}{4} \sum_i \hat{x}_i^2 \approx 1 \quad \Rightarrow \quad \text{RMS}(\hat{x}) \approx 1.$$

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Normalization supplies a stable coordinate system; it does not force the learned output RMS to remain 1.

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inference may process one example or
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an example's prediction changing
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`model.eval()` selects running statistics; “inference is always unbatched” is not the rule.

LayerNorm avoids batch dependence

LayerNorm computes one mean and variance from each example's feature vector:

$$\mu_i = \frac{1}{D} \sum_j x_{ij}, \quad \sigma_i^2 = \frac{1}{D} \sum_j (x_{ij} - \mu_i)^2.$$

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property	BatchNorm	LayerNorm
normalizes over	examples per feature	features per example
depends on batch-mates?	training: yes	no
train / inference stats	batch / running	same computation
common home	CNN feature channels	sequence/token models

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Choose the axis from the invariance you need, not from the layer's name.

GAUGE LEDGER · SAME FIVE-LAYER ANCHOR

control	evidence / status · squared q, g ; RMS r, s where shown
activation	ReLU gate contributes $\frac{1}{2}$
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The reset is defined over the selected axis; learned γ, β may deliberately move the post-normalization RMS.

**Residual connections shorten
the route**

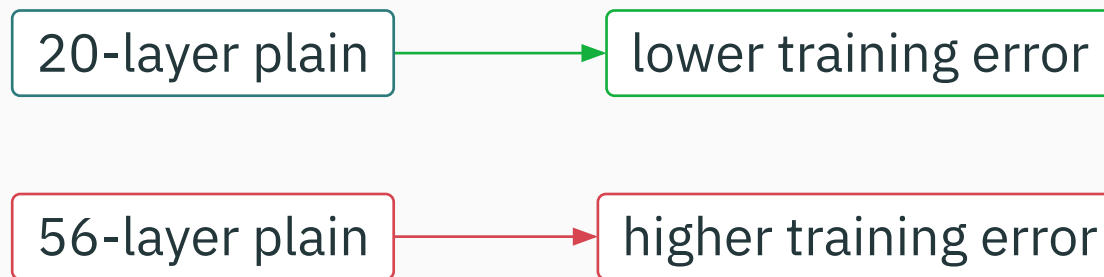


Greater depth can make the training error worse

In the original ResNet experiment on CIFAR-10, a 56-layer **plain** network had higher training error than a 20-layer plain network.

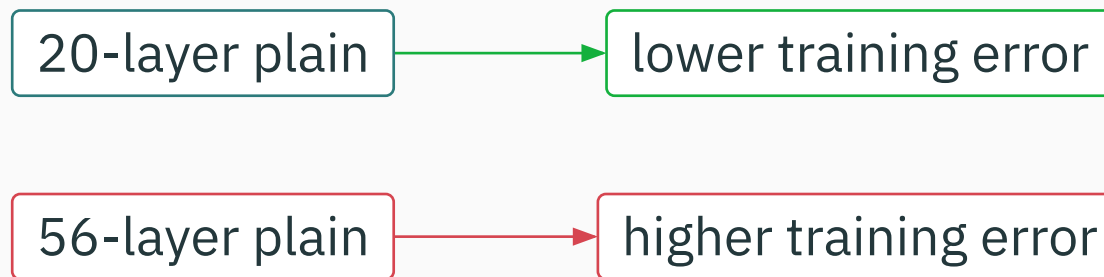
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He et al., “Deep Residual Learning for Image Recognition,” CVPR 2016, Fig. 1.

Learning a correction makes “do nothing” easy

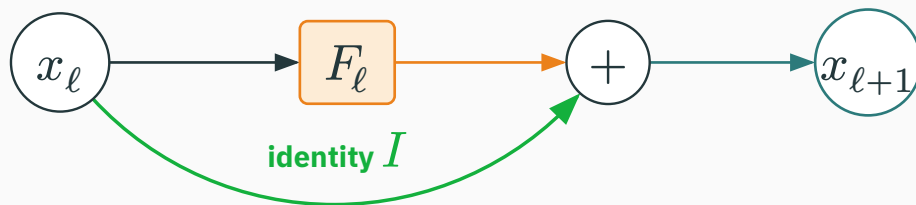
Reparameterize a desired map H_ℓ as

$$F_{\ell(x)} := H_{\ell(x)} - x \quad \Rightarrow \quad H_{\ell(x)} = x + F_{\ell(x)}.$$

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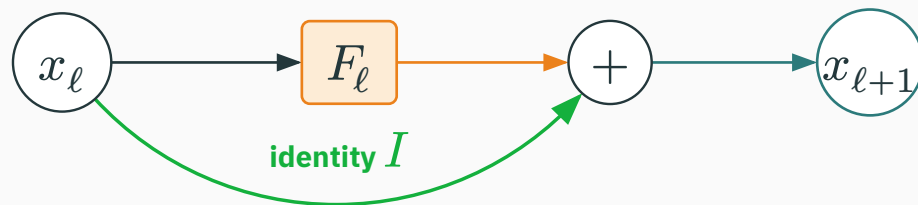
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If the useful transformation is close to identity, the branch only has to learn the small correction F_ℓ .

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Let $J_{F_\ell} = \partial \frac{F_\ell(x_\ell)}{\partial} x_\ell$ and store gradients as column vectors.

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$$\nabla_{x_\ell} \mathcal{L} = \left(I + J_{F_\ell} \right)^T \nabla_{x_{\ell+1}} \mathcal{L}$$

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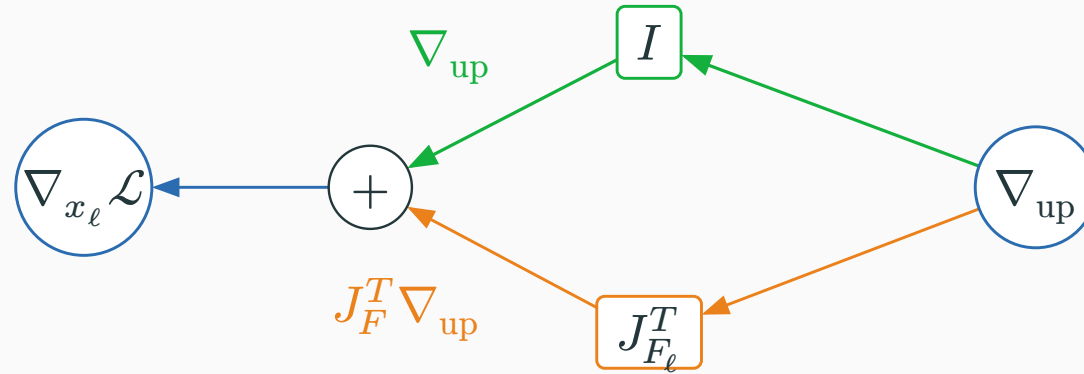
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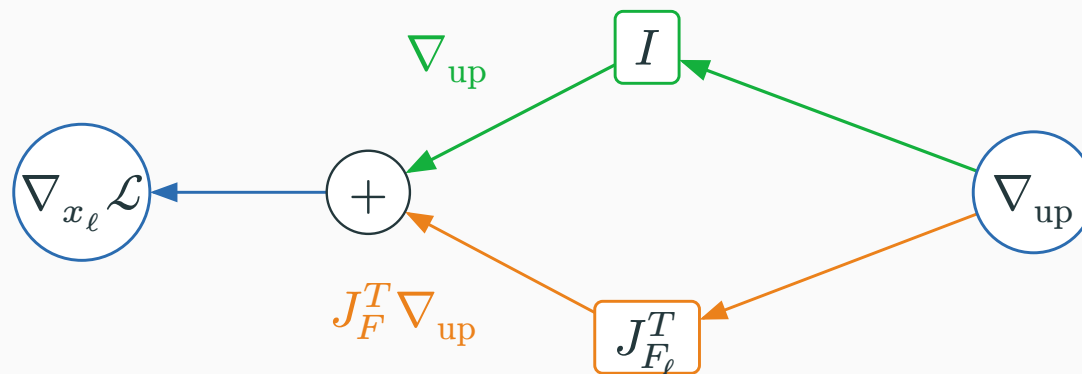
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Backward applies the transposed local Jacobian; the two contributions add at the block input.

The arriving gradient splits into two contributions



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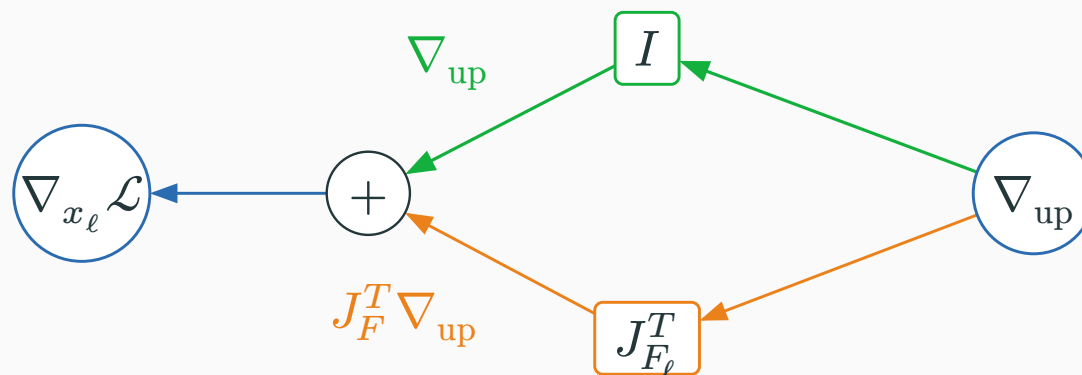
IDENTITY

RESIDUAL

∇_{up} arrives without a branch matrix multiply

$J_F^T \nabla_{up}$ may reinforce or cancel it

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A shortcut adds a route; it does not guarantee that every total gradient is large.

Let one scalar branch have $F'(x) = -0.1$ and let the top gradient magnitude be 1.

After five blocks, which pair of input-gradient magnitudes is correct?

A. plain 0.5; residual 1.5

B. plain 10^{-5} ; residual 0.9^5

C. plain 0.1; residual 0.9

D. plain 0; residual 1 exactly

Answer: the shortcut changes the local multiplier

A ANSWER

path	one block	five-block magnitude	squared gauge
plain branch F	$ -0.1 $	$ -0.1 ^5 = 0.00001$	10^{-10}
residual $I + F$	$ 1 - 0.1 = 0.9$	$0.9^5 = 0.59049$	$0.59049^2 = 0.34868$

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Correct: B — the residual RMS magnitude is 0.59049; its squared ledger is 0.3487.

Residual structure improves the route, not every ingredient

failure	why shortcut alone is insufficient	check
branch explodes	J_F can dominate the identity	branch/output RMS
branch cancels identity	$J_F \approx -I$ in a direction	identity vs residual contribution
shape changes	x and $F(x)$ cannot be added	projection shape/stride
training diverges	learning rate still controls update size	update-to-weight RMS

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Initialization, normalization, residual scaling, and learning rate remain coupled design choices.

GAUGE LEDGER · SAME FIVE-LAYER ANCHOR

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activation	ReLU gate contributes $\frac{1}{2}$
initialization	He gives $q_5 = g_0 = 1$ at $t = 0$
normalization	$\mu = 2.5, \sigma^2 = 1.25$, normalized RMS = 1
residual path	$F' = -.1$: plain RMS 10^{-5} ; residual RMS .59049 ($g = .3487$)

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Each intervention answers a different reason the gauges can drift.

**Measure the system, not the
slogan**

One ledger connects all four controls



control	mechanism	anchor evidence	what it does not guarantee
activation	local gate	ReLU contributes $\frac{1}{2}$	no dead units
initialization	starting gain	He changes $c : .5 \rightarrow 1$	scale later in training
normalization	axis-wise reset	$[1, 2, 3, 4] \rightarrow \text{RMS} = 1$	post- γ, β RMS
residual	short additive route	$10^{-5} \rightarrow .59049$	no cancellation/explosion

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Use the ledger to explain a mechanism, then measure the actual network.

Diagnose with symptom → suspect → test

symptom	first suspect	smallest informative test
activation RMS shrinks by depth	gate/init mismatch	log pre/post RMS by layer
early gradient RMS is tiny	saturation or long path	log gradient RMS and shortcut terms
many inactive ReLU units	bias/update/input scale	active-example count per unit
train/eval predictions differ	BatchNorm statistics	same input in both modes
first non-finite value	scale or update explosion	locate first bad layer and step

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Change one cause-relevant control at a time so the experiment can identify the cause.

Measure the same RMS gauges in code

I INTERACTIVE

```
def rms(x):  
    return x.detach().float().square().mean().sqrt().item()  
  
for name, h in activations.items():  
    print(name, "activation_rms", rms(h))  
  
for name, p in model.named_parameters():  
    if p.grad is not None:  
        print(name, "gradient_rms", rms(p.grad),  
              "finite", bool(p.grad.isfinite().all()))
```

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```

`square().mean().sqrt()` matches r_ℓ, s_ℓ ; a raw `norm()` would grow with width and break the ledger.

Three labs, one predict → run → explain loop

I INTERACTIVE

1 · autopsy	01_signal_gradient_autopsy.ipynb
2 · propagate	02_variance_propagation.ipynb
3 · compare	03_xavier_vs_he.ipynb

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For each notebook: **commit to a trace → run → compare with the ledger → explain any mismatch.**

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For each notebook: **commit to a trace → run → compare with the ledger → explain any mismatch.**

| The three filenames above are direct Colab links to the public repository.

Which statement is defensible without extra assumptions?

A. He exactly preserves every layer's variance throughout training

B. BatchNorm inference must contain only one example

C. Residual shortcuts guarantee gradients cannot vanish

D. Each control targets a mechanism; the actual gauges must still be measured

Correct: D — Each control targets a mechanism; the actual gauges must still be measured

Why: He is an initialization approximation, inference may be batched, and a residual branch can reinforce or cancel the identity contribution.

Revisit your opening bet

ANCHOR · five width-4 layers · ReLU after every affine map · $q_0 = 1 \cdot g_5 = 1$

choice	five-layer squared gauges	five-layer RMS gauges
Xavier normal / uniform	$q_5 = g_0 = .03125$	$r_5 = s_0 = .17678$
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The winning bet was He because ReLU's half-active gate required twice Xavier's variance.

Exit ticket: prescribe the smallest repair

For a 30-layer ReLU MLP, you observe activation RMS $1.0 \rightarrow 0.6 \rightarrow 0.35 \rightarrow \dots$ and early-layer gradient RMS near zero.

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Write three lines:

1. **symptom:** which gauge drifts, and in which direction?
2. **suspect:** which local multiplier could explain it?
3. **test:** which single controlled change or measurement distinguishes the cause?

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A strong answer names the activation/init pair first, verifies per-layer RMS, then considers normalization or a shorter residual route.

Depth is trainable when signal and gradient RMS remain usable along the routes that matter.

Next: L7 asks whether a trainable network also **generalizes**—regularization and the train/test gap.

Further variants and provenance

RMSNorm resets magnitude without mean subtraction

For one example with D features,

$$\text{RMS}(x) = \sqrt{\frac{1}{D} \sum_{j=1}^D x_j^2 + \varepsilon}, \quad y_j = \gamma_j \frac{x_j}{\text{RMS}(x)}.$$

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KEEPS

feature-wise learned scale and RMS
denominator

DROPS

mean subtraction and learned shift in
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the basic form

$\varepsilon > 0$ prevents division by zero and limits amplification when the input RMS is extremely small.

Projection shortcuts can change width and resolution

If $F_{\ell(x_\ell)}$ and x_ℓ have different shapes, use

$$x_{\ell+1} = P_\ell x_\ell + F_{\ell(x_\ell)}.$$

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- stride 1 can change channel count;
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- stride > 1 can change channels **and** downsample spatial resolution.

The shortcut must match shape before addition; it need not be a literal identity tensor.

Placement decides whether the shortcut is truly clean

POST-ACTIVATION

$x_{\ell+1} = \varphi(x_{\ell} + F_{\ell(x_{\ell})})$; the final φ' gates both paths

PRE-ACTIVATION

norm and activation live in F_{ℓ} ; the additive route stays unobstructed

Placement decides whether the shortcut is truly clean

POST-ACTIVATION

$x_{\ell+1} = \varphi(x_{\ell} + F_{\ell}(x_{\ell}))$; the final φ' gates both paths

PRE-ACTIVATION

norm and activation live in F_{ℓ} ; the additive route stays unobstructed

Fixup is a separate carefully scaled initialization strategy that trains deep residual networks without normalization; it is evidence that these controls are coupled, not mandatory in one fixed recipe.

BatchNorm for convolution reduces over B, H, W

For $X \in \mathbb{R}^{B \times C \times H \times W}$, BatchNorm computes one mean and variance per channel:

μ_c, σ_c^2 over $B \cdot H \cdot W$ values of channel c .

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At inference the frozen per-channel running statistics are used, regardless of inference batch size.

Generalized He gain handles a leaky slope

For $\varphi(u) = \max(u, au)$ with a symmetric pre-activation,

$$\mathbb{E}[\varphi(z)^2] \approx \frac{1 + a^2}{2} \mathbb{E}[z^2].$$

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negative slope a	gain factor	$\text{Var}(w)$
0	$\frac{1}{2}$	$\frac{2}{n_{\text{in}}}$
0.1	0.505	$\frac{2}{1.01n_{\text{in}}}$
1	1	$\frac{1}{n_{\text{in}}}$

Primary references and provenance

- Glorot & Bengio (2010), Understanding the difficulty of training deep feedforward neural networks.
- He et al. (2015), Delving Deep into Rectifiers.
- Ioffe & Szegedy (2015), Batch Normalization.
- Ba, Kiros & Hinton (2016), Layer Normalization.
- He et al. (2016), Deep Residual Learning for Image Recognition.
- Zhang & Sennrich (2019), Root Mean Square Layer Normalization.

The activation catalogue and seeded signal-flow trace are generated by `lecture6/diagrams/l6_figs.py`; other diagrams and numeric traces are native Typst vectors computed from the equations shown.