Weighted least squares

$$\hat{\theta} = \underset{\theta}{\arg\min} \sum_{i=1}^{N} d_i \left( y_i - x_i^T \theta \right)^2$$

# Weighted least squares

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \sum_{i=1}^{N} d_i \left(y_i - x_i^T \theta\right)^2$$

$$J(\theta) = \sum_{i=}^{N} d_i \left(y_i - x_i^T \theta\right)^2$$

$$= (y - X\theta)^T D (y - X\theta) \quad ; \quad D = \begin{bmatrix} d_1 & 0 & \cdots & \\ 0 & d_2 & \cdots & \\ & & \ddots & \\ & & & d_N \end{bmatrix}$$

# Weighted least squares

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \sum_{i=1}^{N} d_i \left(y_i - x_i^T \theta\right)^2$$

$$J(\theta) = \sum_{i=}^{N} d_i \left(y_i - x_i^T \theta\right)^2$$

$$= (y - X\theta)^T D (y - X\theta) \quad ; \quad D = \begin{bmatrix} d_1 & 0 & \cdots & \\ 0 & d_2 & \cdots & \\ & & \ddots & \\ & & & d_N \end{bmatrix}$$

$$= (y^T - \theta^T X^T)(Dy - DX\theta)$$

$$J(\theta) = y^T D y - 2 y^T D^T X \theta + \theta^T X^T D X \theta$$

# Weighted least squares

$$J(\theta) = y^T D y - 2 y^T D^T X \theta + \theta^T X^T D X \theta$$

$$\frac{\partial J(\theta)}{\partial \theta} = 0 \Rightarrow \frac{\partial}{\partial \theta} \left(-2 y^T D^T X \theta\right) + \frac{\partial}{\partial \theta} \theta^T X^T D X \theta = \vec{0}$$

$$\Rightarrow 2 X^T D y = 2 X^T D X \theta$$

or $$\boxed{\hat{\theta} = (X^T D X)^{-1} X^T D y}$$