K-Nearest Neighbors

Nipun Batra

July 26, 2025

IIT Gandhinagar

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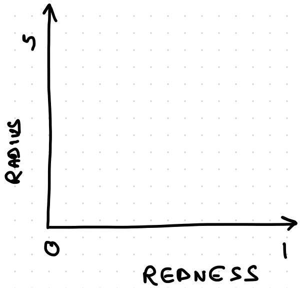
Introduction and Fundamentals

CLASSIFICATION



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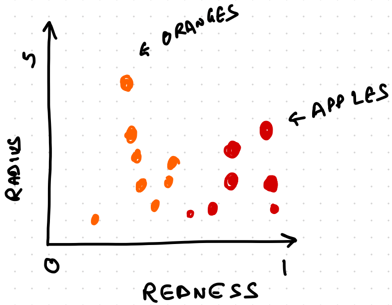

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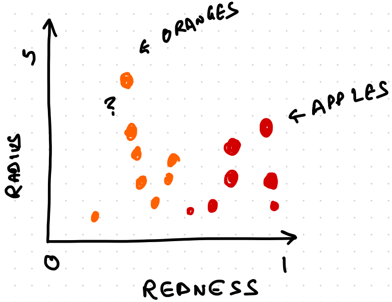

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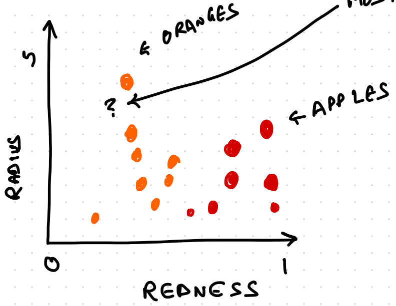
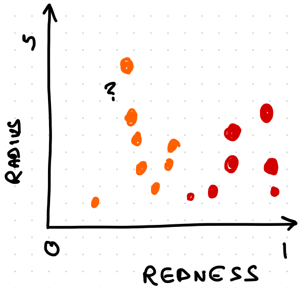


MOST LIKELY AN ORANGE ∵ "SIMILAR" TO ORANGES IN ATTRIBUTES

CLASSIFICATION



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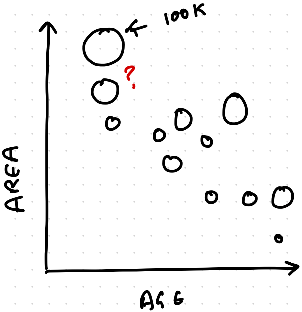

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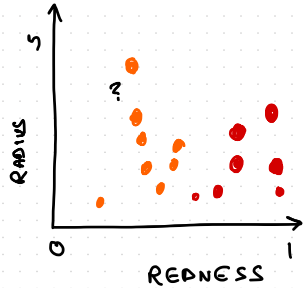

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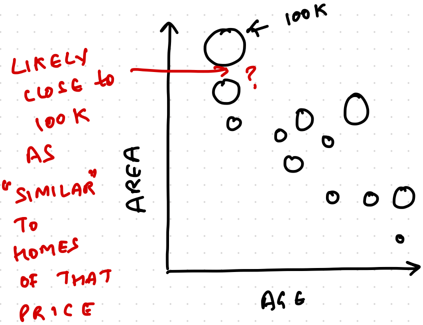
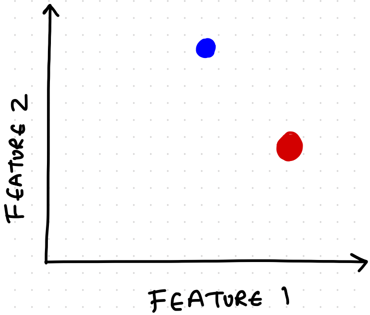


VORONOI DIAGRAM FOR 1-NN



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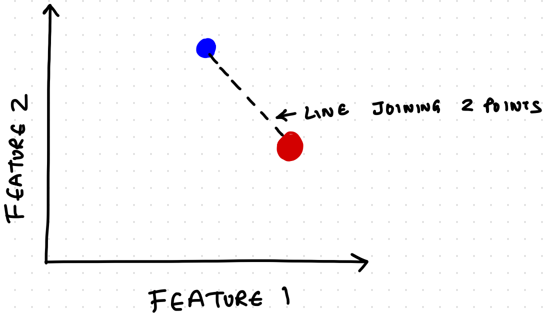

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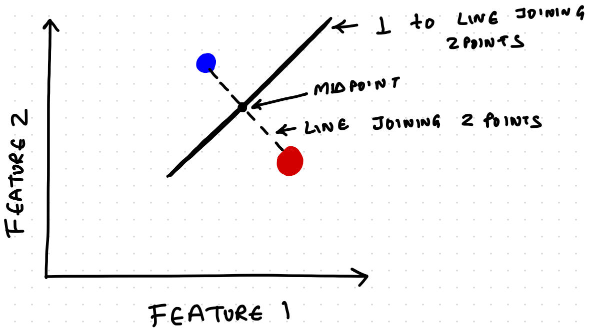

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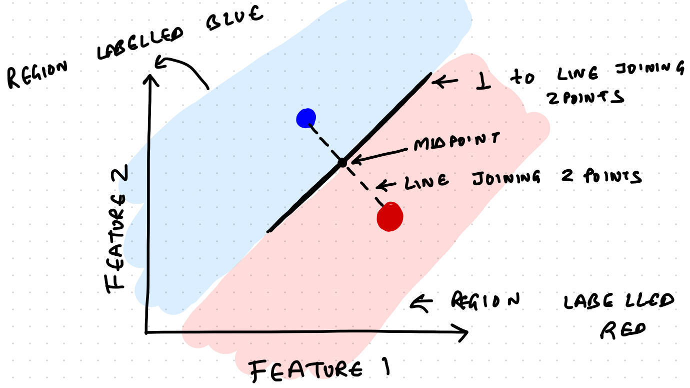

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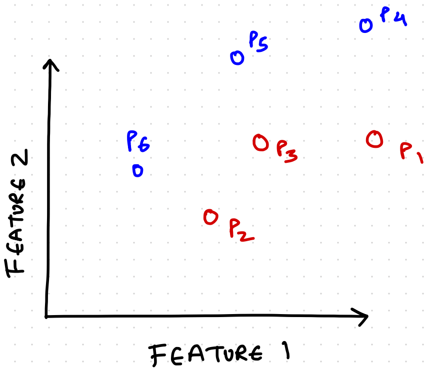

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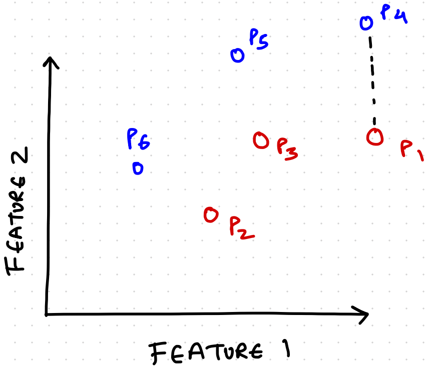
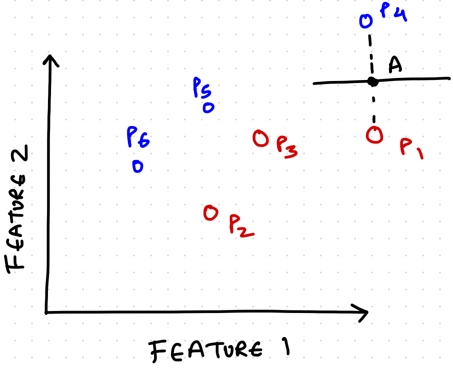


VORONOI DIAGRAM FOR 1-NN

A: MID PT B/W P1 & P4



VORONOI DIAGRAM FOR 1-NN

A: MID PT B/W P_1 & P_4

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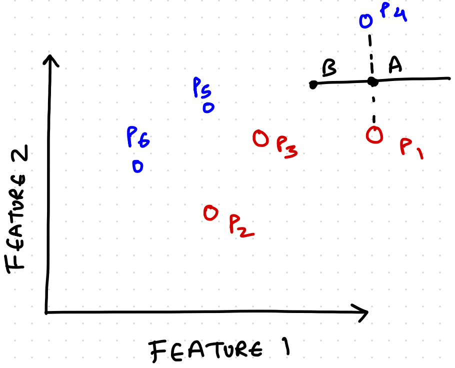

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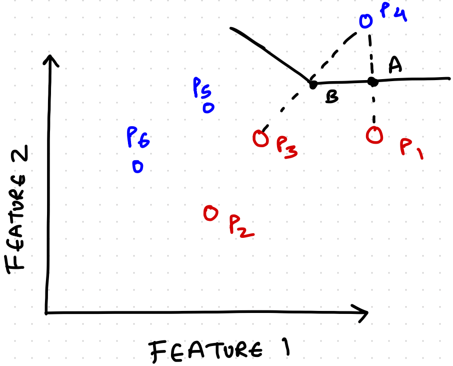
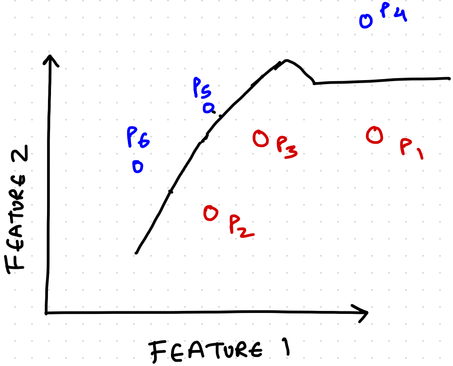


VORONOI DIAGRAM FOR 1-NN

DECISION BOUNDARY IS PIECE-WISE LINEAR



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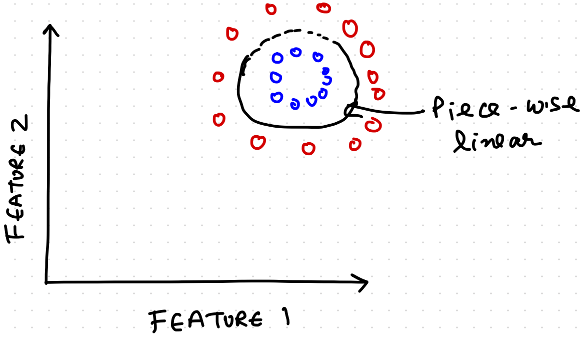
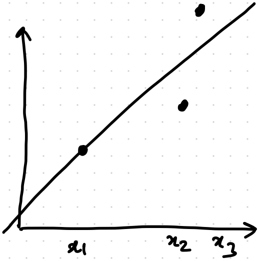


LINEAR REGRESSION



1NN REGRESSION

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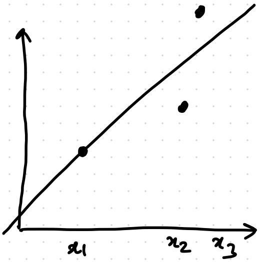

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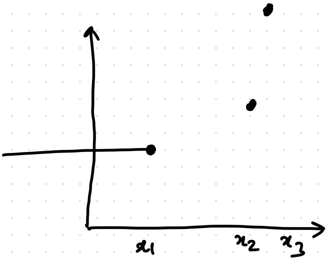

$x < x_1$: NN is (x_1, y_1)

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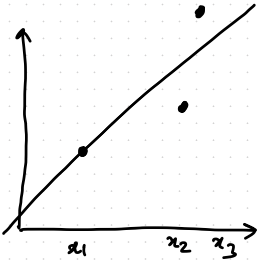

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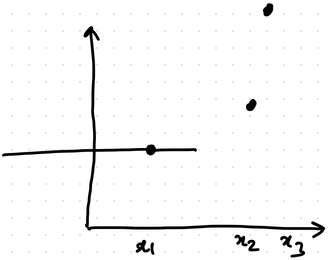

$x < x_1$: NN is (x_1, y_1)

$x < \frac{x_1+x_2}{2}$: NN is (x_1, y_1)

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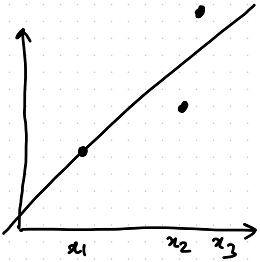

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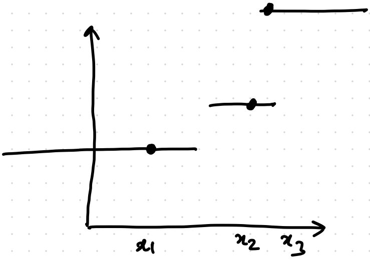

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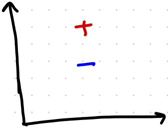
$x < \frac{x_1 + x_2}{2}$: NN is (x_1, y_1)

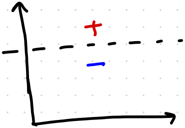
$\frac{x_1 + x_2}{2} < x < \frac{x_2 + x_3}{2}$: NN is (x_2, y_2)

KNN IS NON-PARAMETRIC



LINEAR MODEL

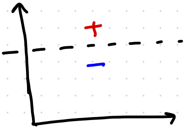
KNN IS NON-PARAMETRIC



LINEAR MODEL

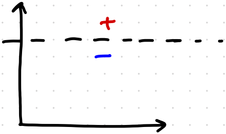
DECS^n BOUNDARY $y = mx + c$ (# params = 2)

KNN IS NON-PARAMETRIC



LINEAR MODEL

DECS^{n} BOUNDARY $y = mx + c$ (# params = 2)



KNN (K=1)

DECS^{n} BOUNDARY (LIKE $y = mx + c$)

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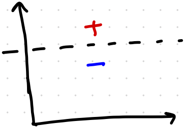

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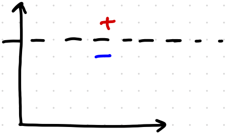

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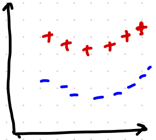
ADD DATA



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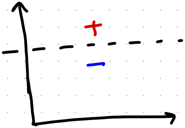

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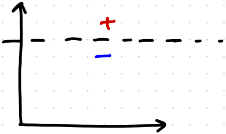

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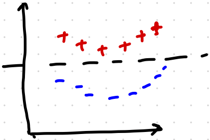
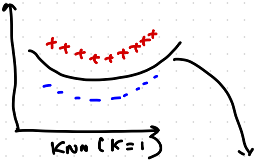


LINEAR MODEL

DECS^N BOUNDARY $y = mx + c$ (2 params)



KNN (K=1)

PARAMS >> 2 (AT LEAST CUBIC)

PARAMETRIC

PARAMS FIXED WRT DATASET SIZE

MAKE ASSUMPTIONS (LIKE FUNCTIONAL FORM)

USUALLY QUICKER

Eg: LINEAR MODELS, SVM (LINEAR, POLYNOMIAL)

NON-PARAMETRIC

PARAMS GROWS WRT DATASET SIZE

LESSER ASSUMPTIONS

USUALLY SLOWER

Eg: KNN, DT, SVM (with RBF)

Parametric vs Non-Parametric Models

	Parametric	Non-Parametric
Parameter	Number of parameters is fixed w.r.t dataset size	Number of parameters grows w.r.t. to an increase in dataset size
Speed	Quicker (as the number of parameters are less)	Longer (as number of parameters are less)
Assumptions	Strong Assumptions (like linearity in Linear Regression)	Very few (sometimes no) assumptions
Examples	Linear Regression	KNN, Decision Tree

Lazy vs Eager Strategies

	Lazy	Eager
Train Time	0	$\neq 0$
Test	Long (due to comparison with train data)	Quick (as only "parameters" are involved)
Memory	Store/Memorise entire data	Store only learnt parameters
Utility	Useful for online settings	
Examples	KNN	Linear Regression, Decision Tree

Important Considerations

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- What are the **number of neighbors** that you are going to take into consideration?
- What is the **computational complexity** of the algorithm that you are implementing?

Distance Metrics and Considerations

Important Considerations: Distance Metric

The Distance Metric acts as a *measure of similarity* between the points.

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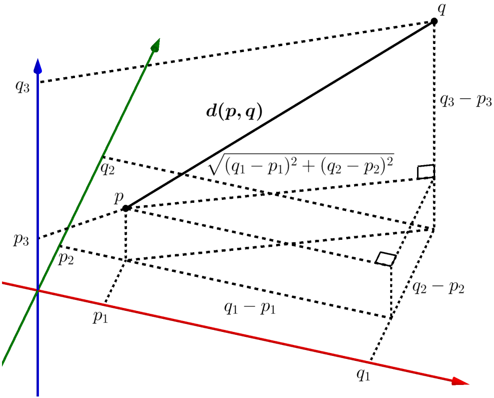

Euclidean Distance

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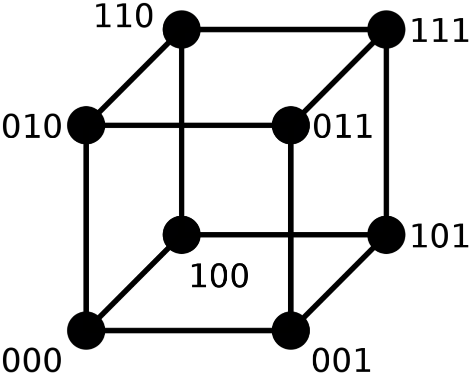

Hamming Distance

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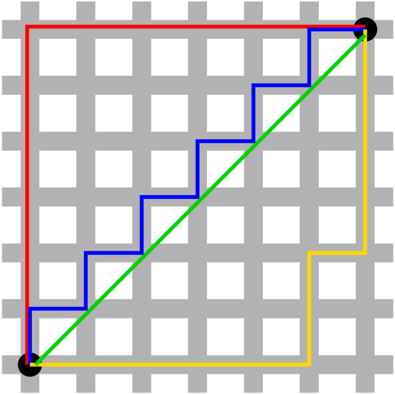

Manhattan Distance

Important Considerations: Value of K

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High values of K will result in smoother decision boundaries $\implies$ lower variance but also higher bias

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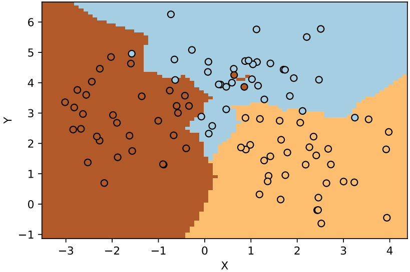

K = 3

Important Considerations: Value of K

Aggregating data

There are different ways to go about aggregating the data from the K nearest neighbors.

- Median

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- Median
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- Mode

KNN Algorithm and Implementation

KNN Algorithm

- Keep the entire dataset: (x, y)

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KNN Algorithm

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  2. Predict y^*

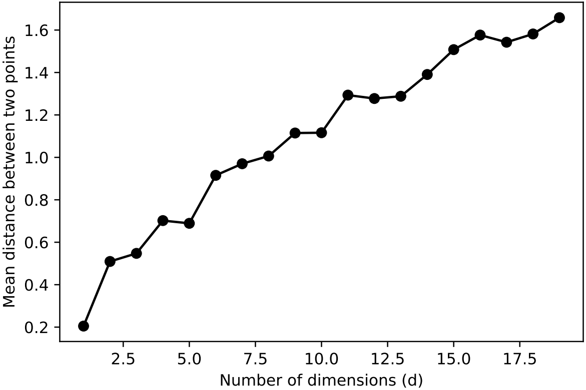
Challenges and Extensions

With an increase in the number of dimensions:

Curse of Dimensionality

With an increase in the number of dimensions:

1. the distance between points starts to increase



For a unifromly random dataset

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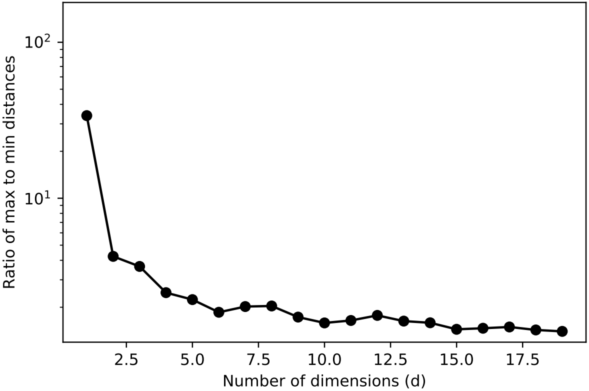

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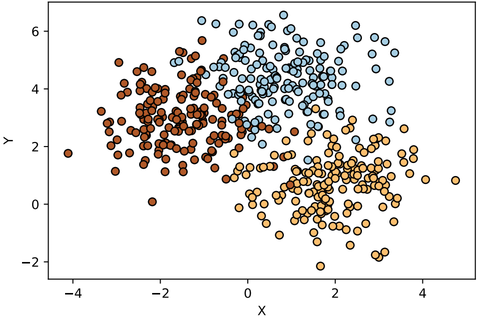

Example of a big dataset

Approximate Nearest Neighbors

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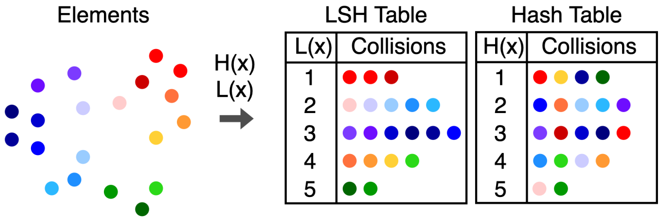
Such techniques include:

- Locality sensitive hashing
- Vector approximation files
- Greedy search in proximity neighborhood graphs

Normal hash functions $H(x)$ try to keep the collision of points across bins uniform.



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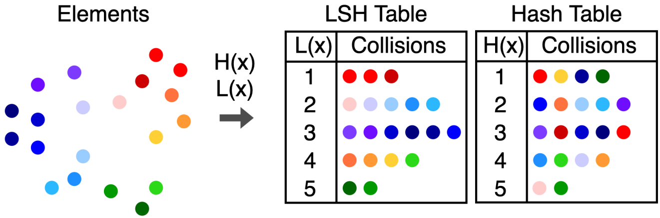
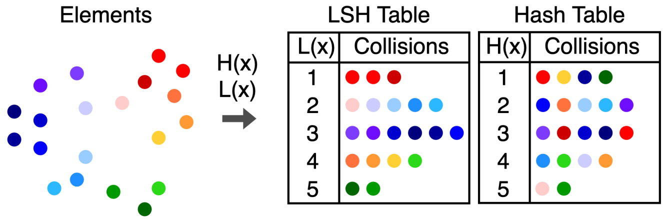


Example of a big dataset

Locality sensitive hashing

A locality sensitive hash (LSH) function $L(x)$ would be designed such that similar values are mapped to similar bins.

For such cases, all elements in a bin would be given the same label, which again can be decided on the basis of different aggregation methods



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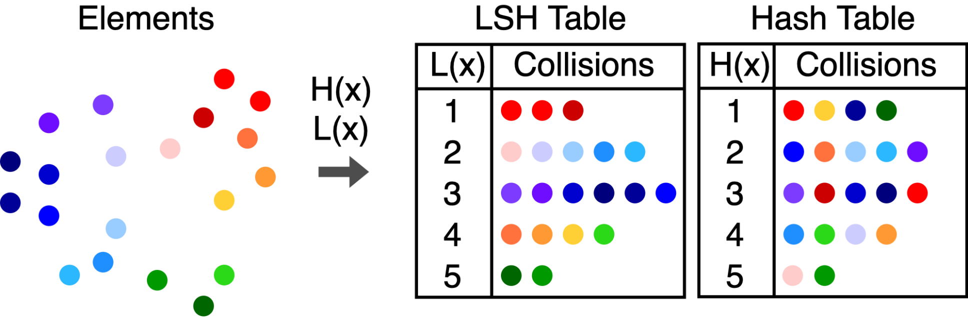
Elements
H(x)
L(x)
LSH Table
L(x)
Collisions
1
2
3
4
5
Hash Table
H(x)
Collisions
1
2
3
4
5

Practice and Summary

Pop Quiz: KNN Concepts

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