Unsupervised Learning

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Places where you will see unsupervised learning

- It can be used to segment the market based on customer preferences.
- A data science team reduces the number of dimensions in a large dataset to simplify modeling and reduce file size.

Clustering

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gt_iris.png

Iris Data Set with ground truth

K-Means Clustering

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k_1.png

Dataset with 5 clusters

K-Means Clustering

k_ex_6.png

K=6

K-Means Clustering

k_ex_6.png

K=6

k_ex_5.png

K=5

K-Means Clustering

k_ex_6.png

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K-Means Clustering

k_ex_6.png

K=6

k_ex_5.png

K=5

k_ex_4.png

K-Means Clustering

k_ex_6.png

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k_ex_3.png

K-Means Intuition

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  Minimize the WCV as much as possible

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$$WCV\left(C_i\right)=\frac{1}{\left|C_i\right|}_{PROTECTED_0}$$

$\text{WCV}(C_i) = \frac{1}{|C_i|} \sum_{a \in C_i} \sum_{b \in C_i} ||x_a - x_b||_2^2$

where $|C_i|$ is the number of points in C_i

K-Means Algorithm

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K-Means Algorithm

1. Randomly assign a cluster number i to every point (where $i \in \{1, \ldots n\}$)
2. Iterate until convergence:
   2.1 For each cluster C_i compute the centroid (mean of all points in C_i over d dimensions)
   2.2 Assign each observation to the cluster which is the closest.

Working of K-Means Algorithm

Why does K-Means work?

$$\begin{aligned}\text{Let, } x_i \in R^d &= \text{Centroid for} i^{th} \text{cluster} \\ &= \frac{1}{|C_i|} \sum_{a \in C_i} x_a\end{aligned}$$

Why does K-Means work?

$$\text{Let, } x_i \in R^d = \text{Centroid for} i^{th} \text{cluster}$$
$$= \frac{1}{|C_i|} \sum_{a \in C_i} x_a$$

Then,

$$WCV\left(C_i\right) = \frac{1}{|C_i|} \sum_{a \in C_i} \sum_{b \in C_i} ||x_a - x_b||_2^2$$
$$= 2 \sum_{a \in C_i} ||x_a - x_i||_2^2$$

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This shows that K-Means gives the **local minima**.

Hierarchal Clustering

Gives a clustering of all the clusters

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There is no need to specify K at the start

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k_bad_1.png

k_bad_2.png

Examples where K-Means fails

Algorithm for Hierarchal Clustering

1. Start with all points in a single cluster

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    2.1 Identify the 2 closest points

h_e_1.png

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1. Start with all points in a single cluster

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    2.2 Merge them

h_e_1.png

Algorithm for Hierarchal Clustering

1. Start with all points in a single cluster
2. Repeat until all points are in a single cluster
   2.1 Identify the 2 closest points
   2.2 Merge them

h_e_1.png

h_e_2.png

Joining Clusters/Linkages

Complete

Max inter-cluster
similarity

Joining Clusters/Linkages

Complete

Max inter-cluster similarity

Single

Min inter-cluster similarity

Joining Clusters/Linkages

Complete
Max inter-cluster similarity

Single
Min inter-cluster similarity

Centroid
Dissimilarity between cluster centroids

More Code

Google Colab Link