COORDINATE DESCENT (ITERATIVE METHOD)

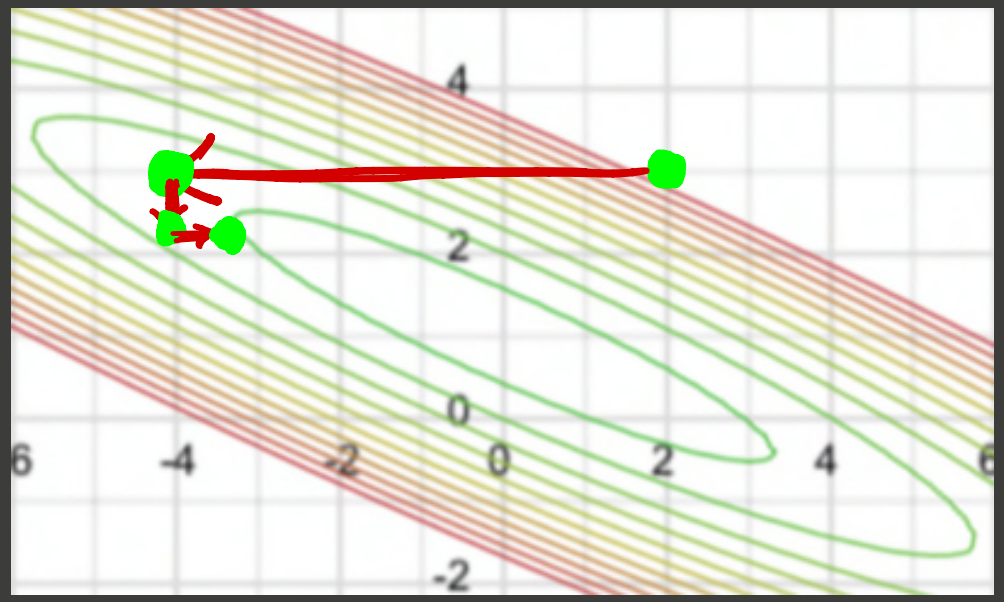
(1) INITIALISE ALL θ_i's

(2) TILL CONVERGENCE

(2.1) CHOOSE DIMENSION $j \in (1, \ldots d)$

(2.2) FIX ALL θ_i's except θ_j

(2.3) MINIMIZE COST $(\theta_1, \ldots \theta_j \ldots \theta_d)$ wrt θ_j



$$\sum e_{y_1}^2 = 14 + 3\theta_0^2 + 14\theta_1^2 - 12\theta_0 - 28\theta_1 + 12\theta_0\theta_1$$

x	y
1	1
2	2
3	3

① INIT $\theta_0 = 2 \cdot \theta_1 = 3$

② I1 FIX $\theta_1 = 3$; OPTIMIZE FOR θ_0

$$\sum e_{y_1}^2(\theta_0) = 3\theta_0^2 - 12\theta_0 + 36\theta_0 + K = 3\theta_0^2 + 24\theta_0 + K$$

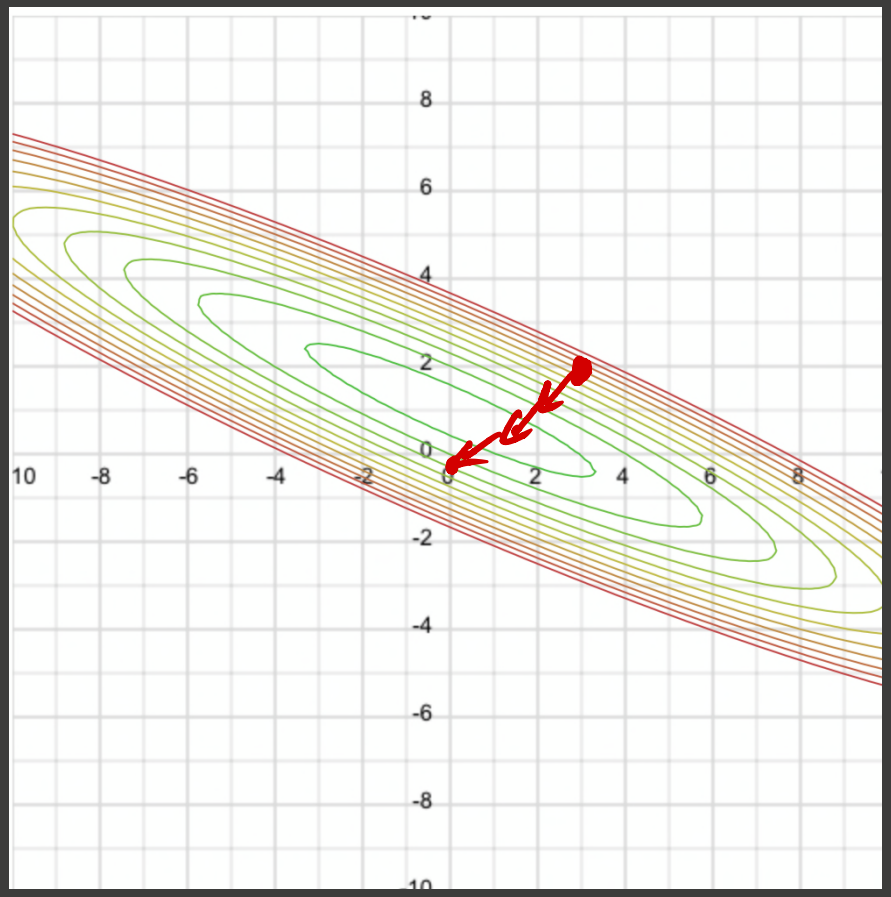
$$\frac{\partial}{\partial\theta_0} \sum e_{y_1}^2(\theta_0) = 0 \Rightarrow 6\theta_0 = -24 \Rightarrow \theta_0 = -4$$

I2 NOW FIX $\theta_0 = -4$, OPTIMIZE FOR θ_1

$\theta_1 \simeq 2.7$

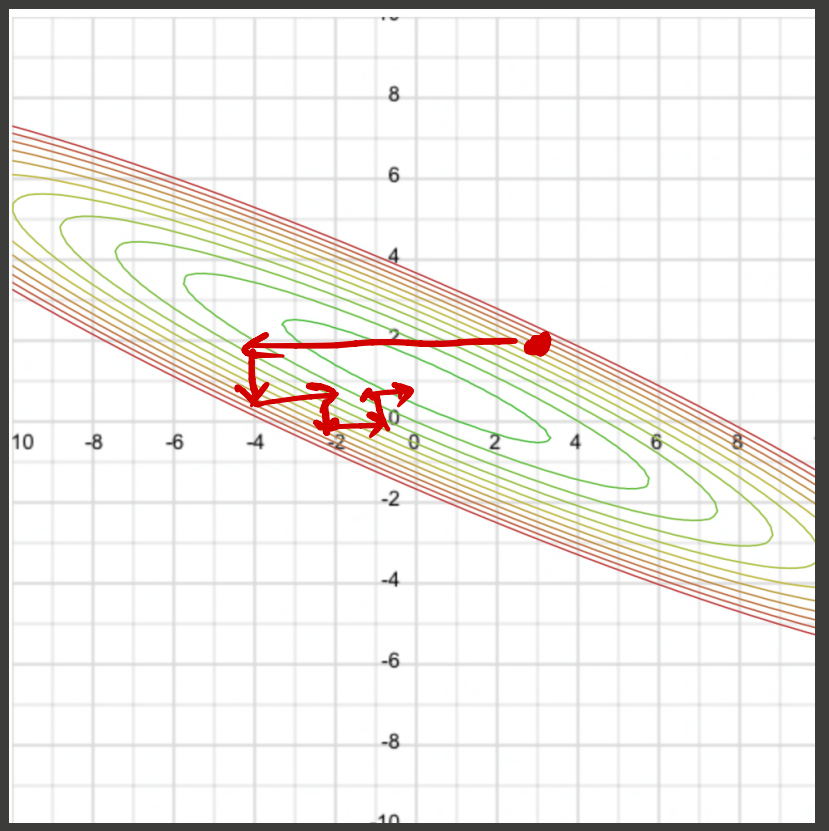
I3 FIX $\theta_1 = 2.7$; OPTIMIZE FOR θ_0 → GIVES US $\theta_0 = -3.4$

GRADIENT DESCENT

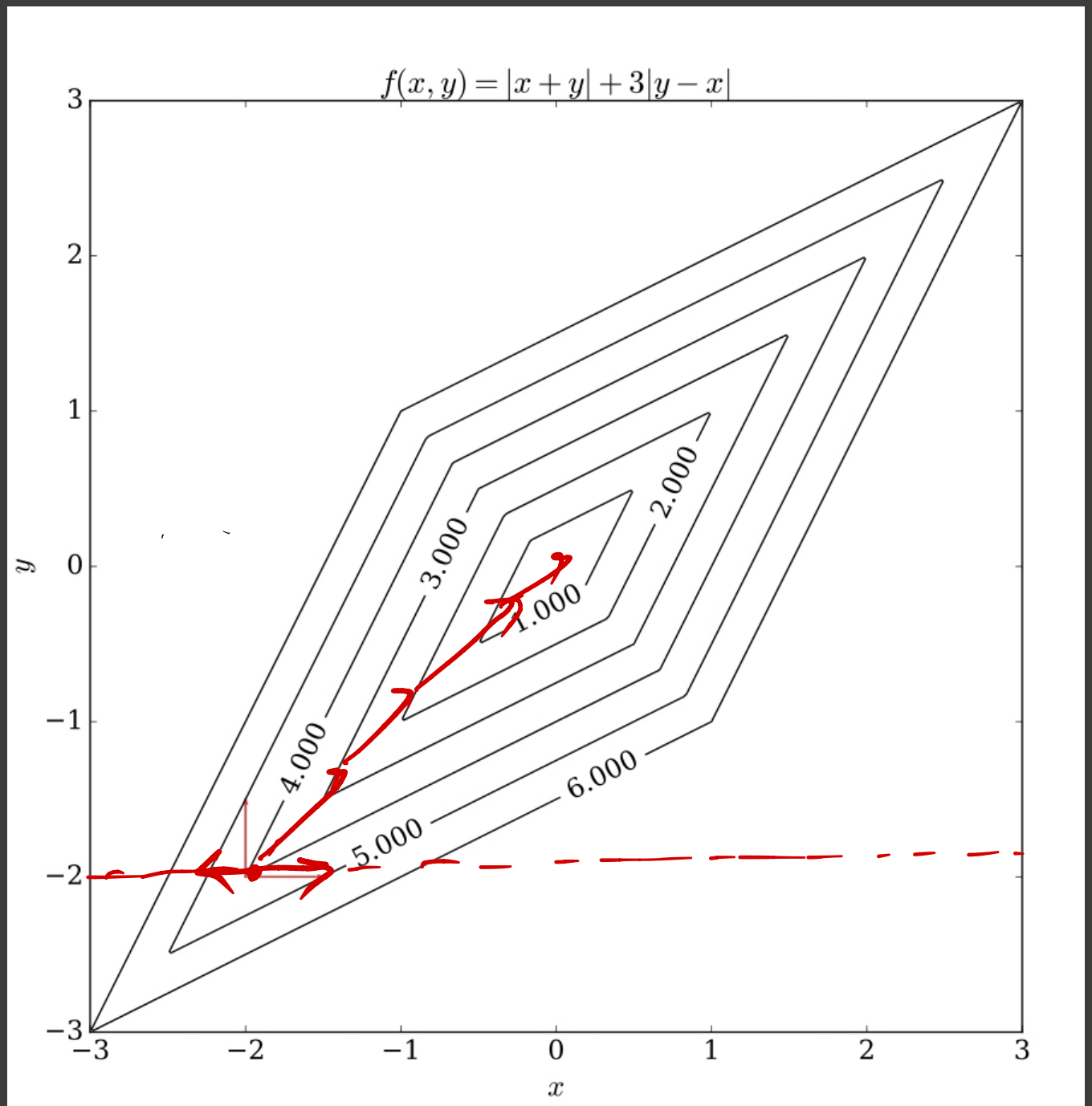


updates in all coordinates / dimensions at a given time

COORDINATE DESCENT



Updates only in '1' coordinate / dimension at a time



WILL COORDINATE DESCENT YIELD CORRECT RESULT?

START WITH $(x, y) = (-2, -2)$

$f(x, y) = |x + y| + 3|y - x|$

Fix 'y'

$f(x) = |x - 2| + 3|x + 2|$

$x < -2$

$f(x) = 2 - x + 3(-x - 2)$

$= -4x - 4$

$\frac{\partial f(x)}{\partial x} = 0 \Rightarrow -4 = 0$

$-2 < x < 2$

$x > 2$

$f(x) = x - 2 + 3x + 6$

$= 4x + 4$