SUPPORT VECTOR MACHINES

* Popular binary classification technique.

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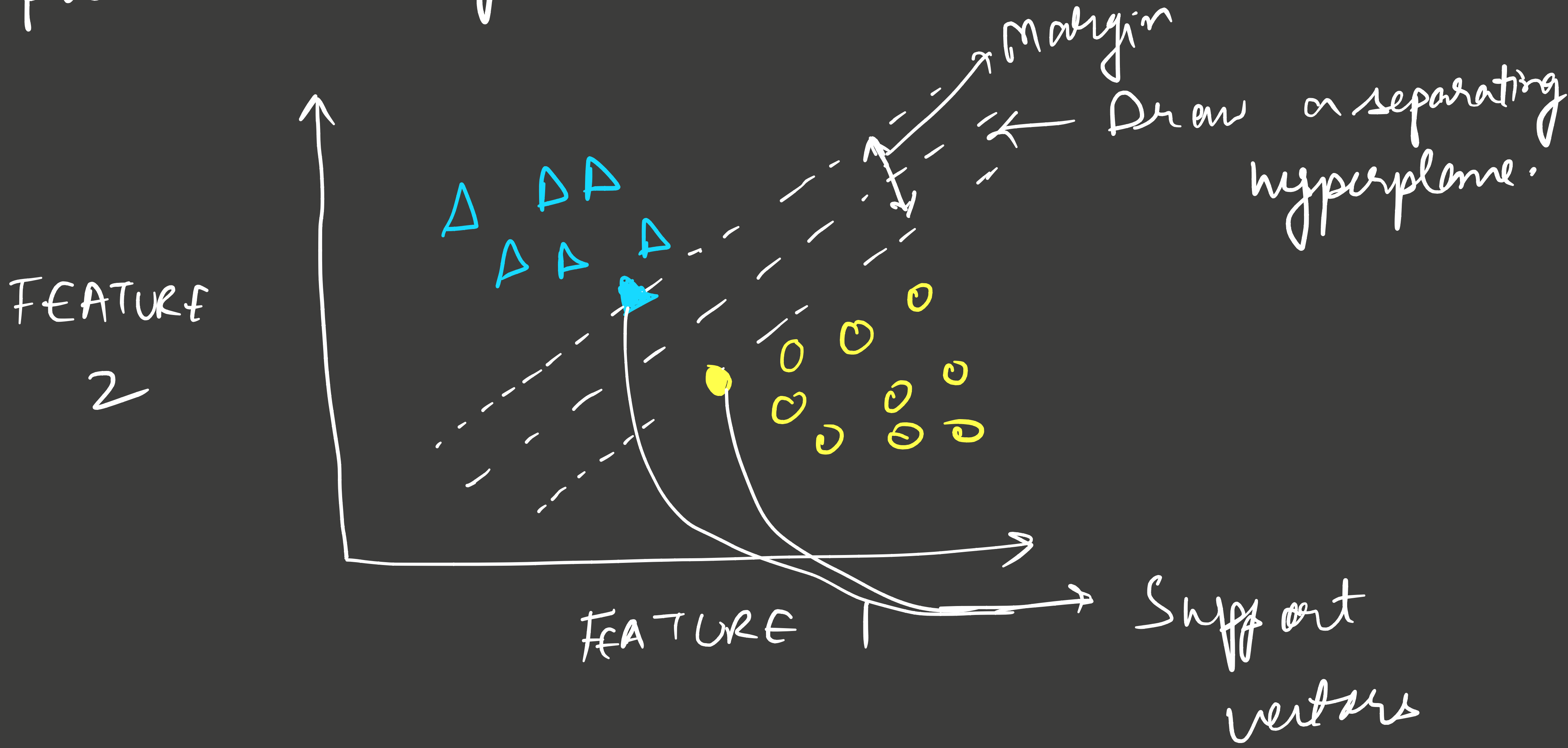
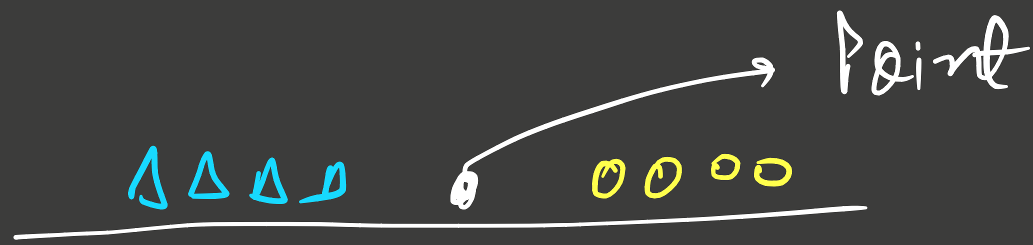


→ Draw a separating hyperplane

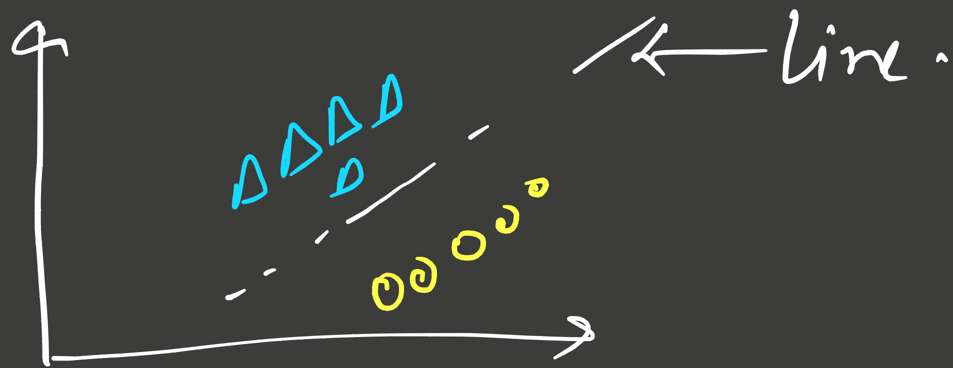
→ Maximize the margin

Hyperplane v/s Dimensions

1D

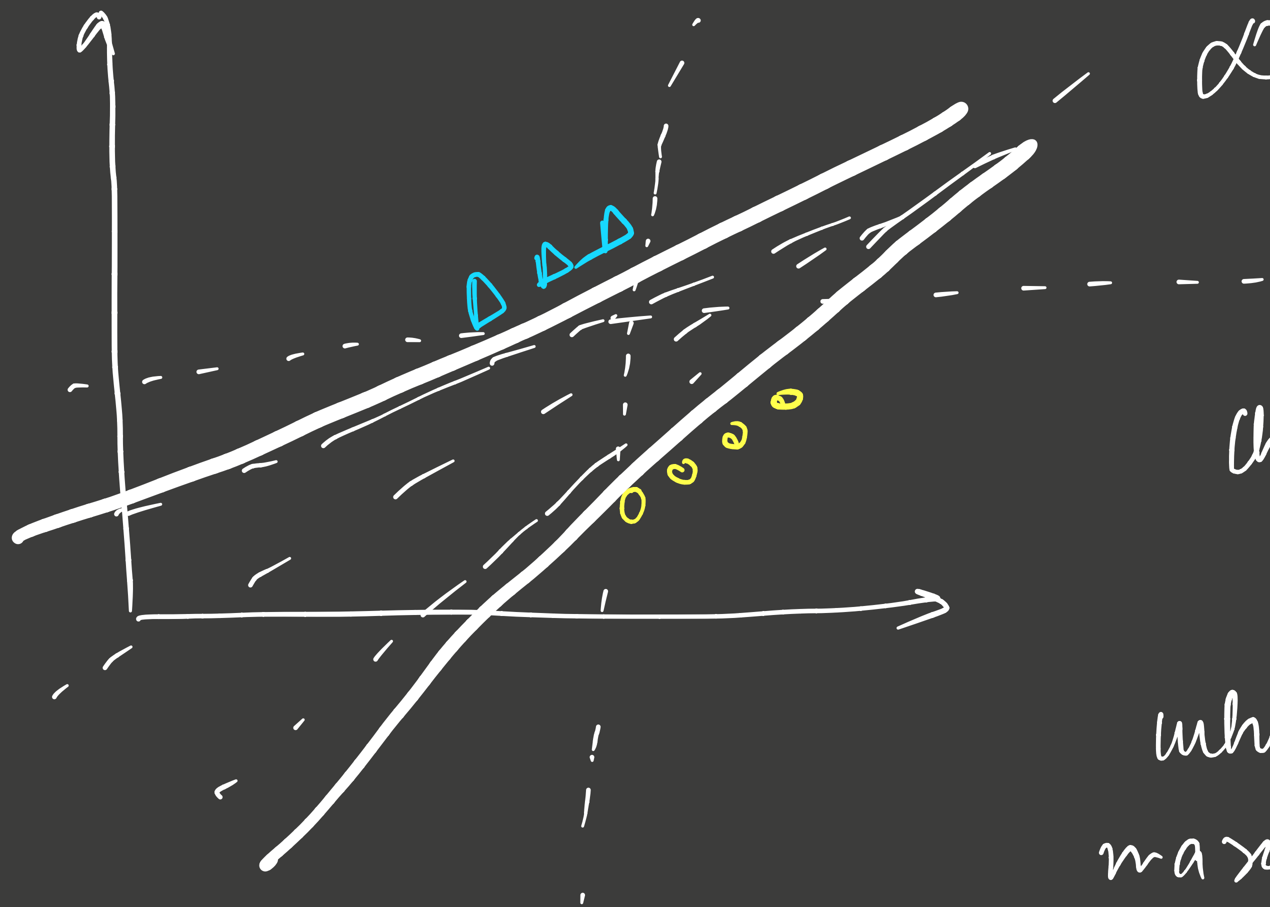


2D



3D
&
above.

How many separating planes?

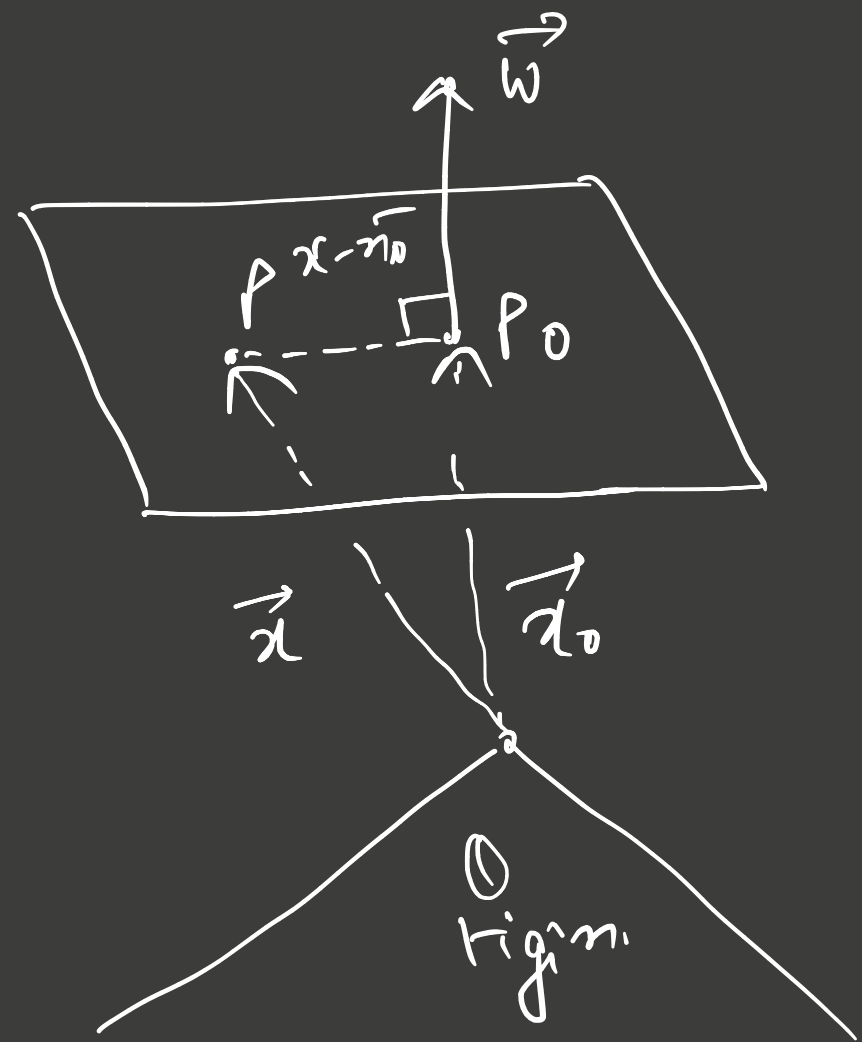


∞ such planes!

Choose the one which maximizes margin

EQUATION OF HYPERPLANE

* Defined by a point (P_0) and $\perp$ vector to that plane at that point ($\vec{w}$)



P & P_0 on plane.

Now

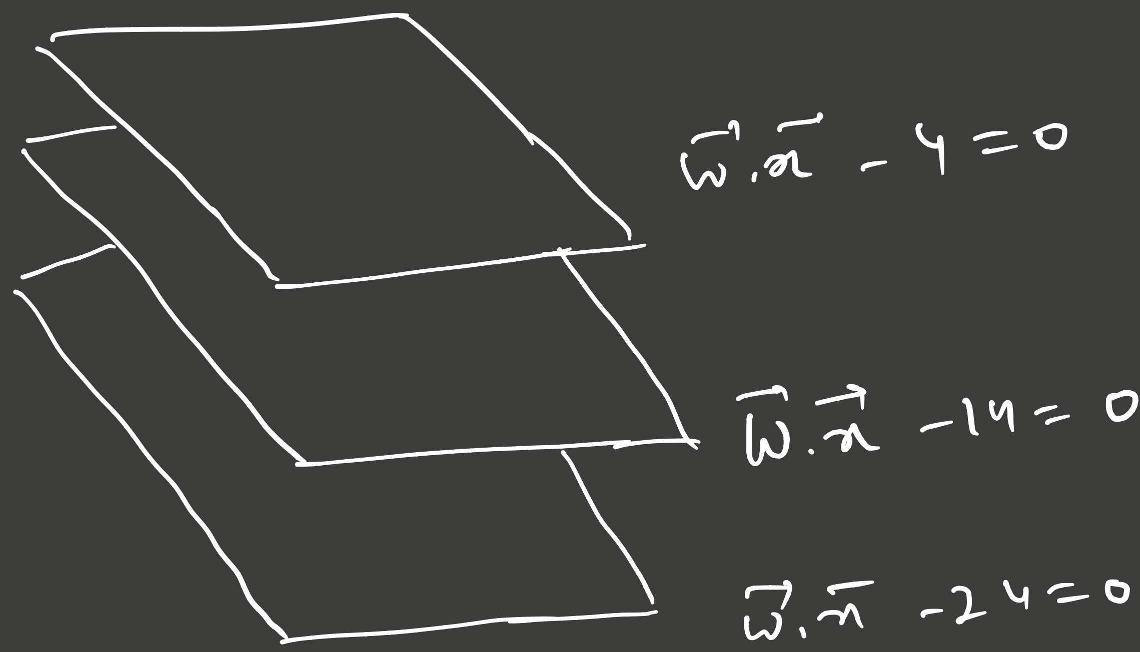
Any point

$\vec{w} \perp \vec{x} - \vec{x}_0$

or $\vec{w} \cdot (\vec{x} - \vec{x}_0) = 0$

or $\vec{w} \cdot \vec{x} - \vec{w}\,\vec{x}_0 = 0$

or $\boxed{\vec{w} \cdot \vec{x} + b = 0}$



$$\vec{w} = (2, 1, 4)$$

$$P_0 = (0, 2, 3)$$

$$b = -\vec{w}\cdot\vec{x_0} = -(2\times 0 + 1 * 2 + 4 * 3) = -(2+12)$$

$$= -14$$

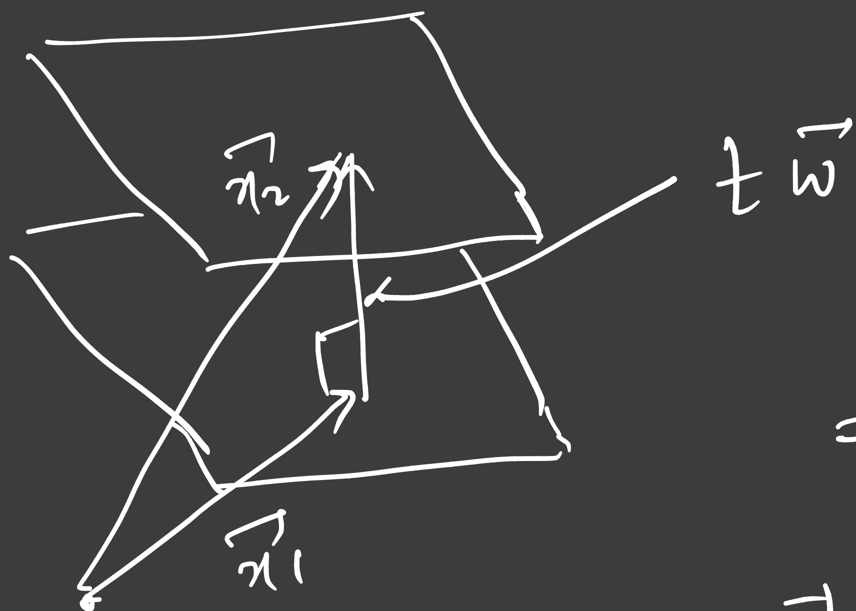
Distance b/w 2 || hyperplanes

① $\vec{w}\cdot\bar{x} + b_1 = 0$

② $\bar{w}\cdot\bar{x} + b_2 = 0$

$$\vec{x_2} = \vec{x_1} + t\vec{w}$$

$$D = |t\vec{w}| = |t|\,\|\vec{w}\|$$



$$\vec{w}\cdot\vec{x_2} + b_2 = 0$$

$$\Rightarrow \vec{w}\cdot(\vec{x_1} + t\vec{w}) + b_2 = 0$$

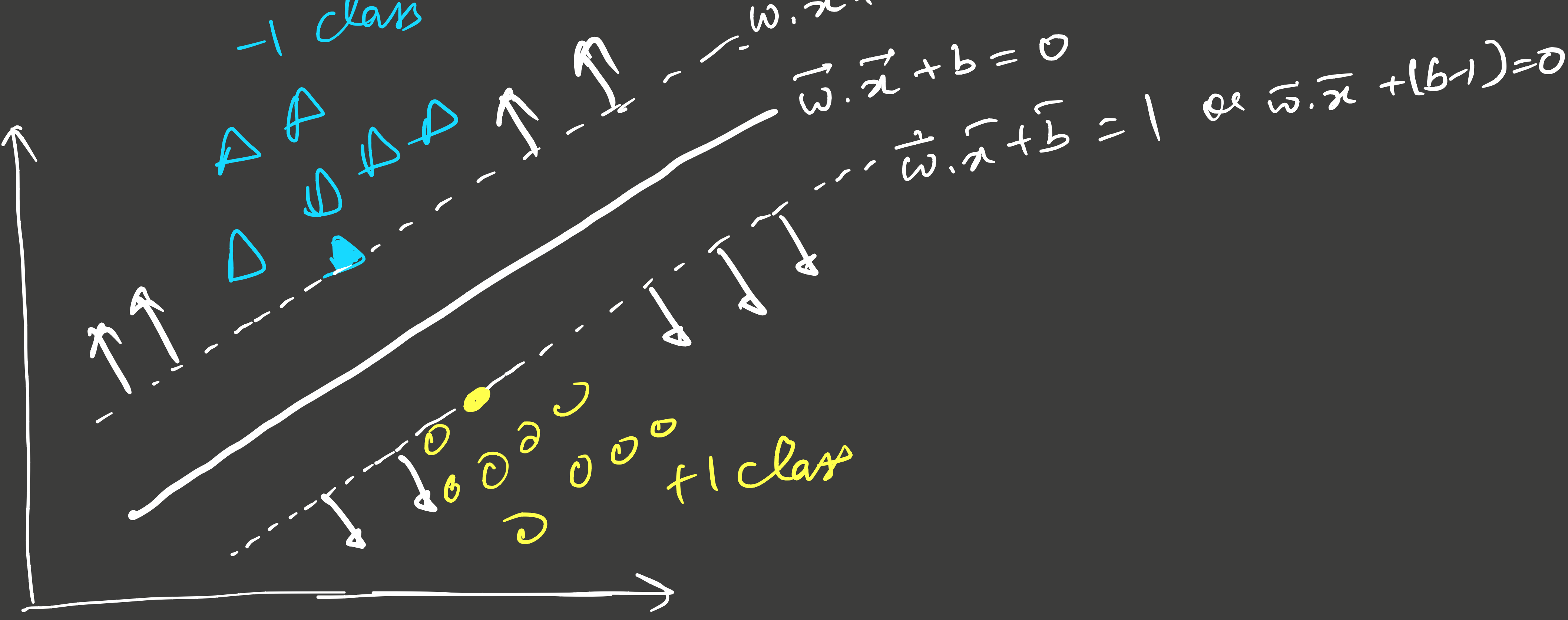
$$\Rightarrow \vec{w}\cdot\vec{x_1} + t\|\vec{w}\|^2 + b_1 - b_1 + b_2 = 0$$

$$\Rightarrow \overset{0}{\cancel{(\vec{w}\cdot\vec{x_1} + b_1)}} + b_2 - b_1 + t\|\vec{w}\|^2 = 0$$

$$\Rightarrow t = \frac{b_2 - b_1}{\|\vec{w}\|^2} \Rightarrow$$

$$\boxed{D = t\|\vec{w}\| = \frac{b_2 - b_1}{\|\vec{w}\|}}$$

FORMULATION



$$\text{Margin} = \frac{(b+1)-(b-1)}{\|\vec{w}\|} = \frac{2}{\|\vec{w}\|}$$

mathematical convenience.

Goal: Maximize margin = Minimize $\|\vec{w}\|$ or $\frac{1}{2}\|\vec{w}\|$

s.t. correctly labelling other points.

i.e. $\vec{w}\cdot\vec{x}+b \le -1$ if $y_i=-1$

$\vec{w}\cdot\vec{x}+b \ge 1$ if $y_i=+1$

PRIMAL FORMULATION

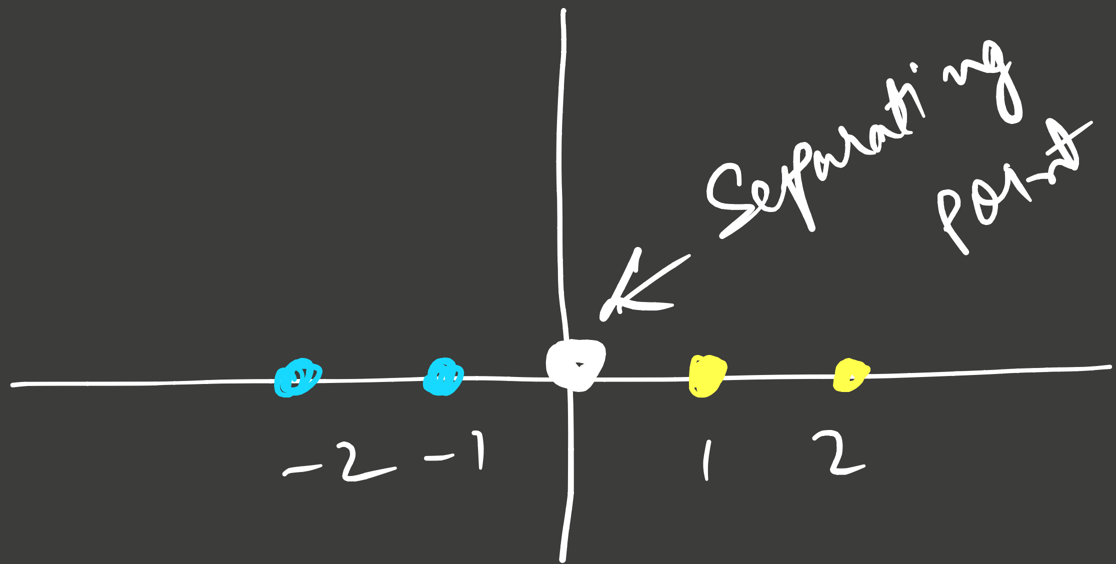
$$\boxed{\begin{aligned} &\text{Minimize} \quad \frac{1}{2}\|\vec{w}\|^2 \\ &\text{s.t.} \quad y_i(\vec{w}\cdot\vec{x_i}+b)\geq 1 \quad \forall i \end{aligned}}$$

Q) what is $\|\vec{w}\|$

$$\vec{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \end{bmatrix} \qquad \|\vec{w}\| = \sqrt{w^T w}$$

$$= \sqrt{\begin{bmatrix} w_1 & w_2 & \cdots \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \end{bmatrix}}$$

SIMPLE EXAMPLE (1D)



4 points

x_1	y
1	1
2	1
-1	-1
-2	-1

Separating hyperplane: $w_1 x_1 + b = 0$

$$y_i (w_1 x_1 + b) \geq 1$$

$$\Rightarrow \quad 1 (w_1 + b) \geq 1 \quad \ldots (1)$$

$$1 (2 w_1 + b) \geq 1 \quad \ldots (2)$$

$$-1 (-w_1 + b) \geq 1 \quad \ldots (3)$$

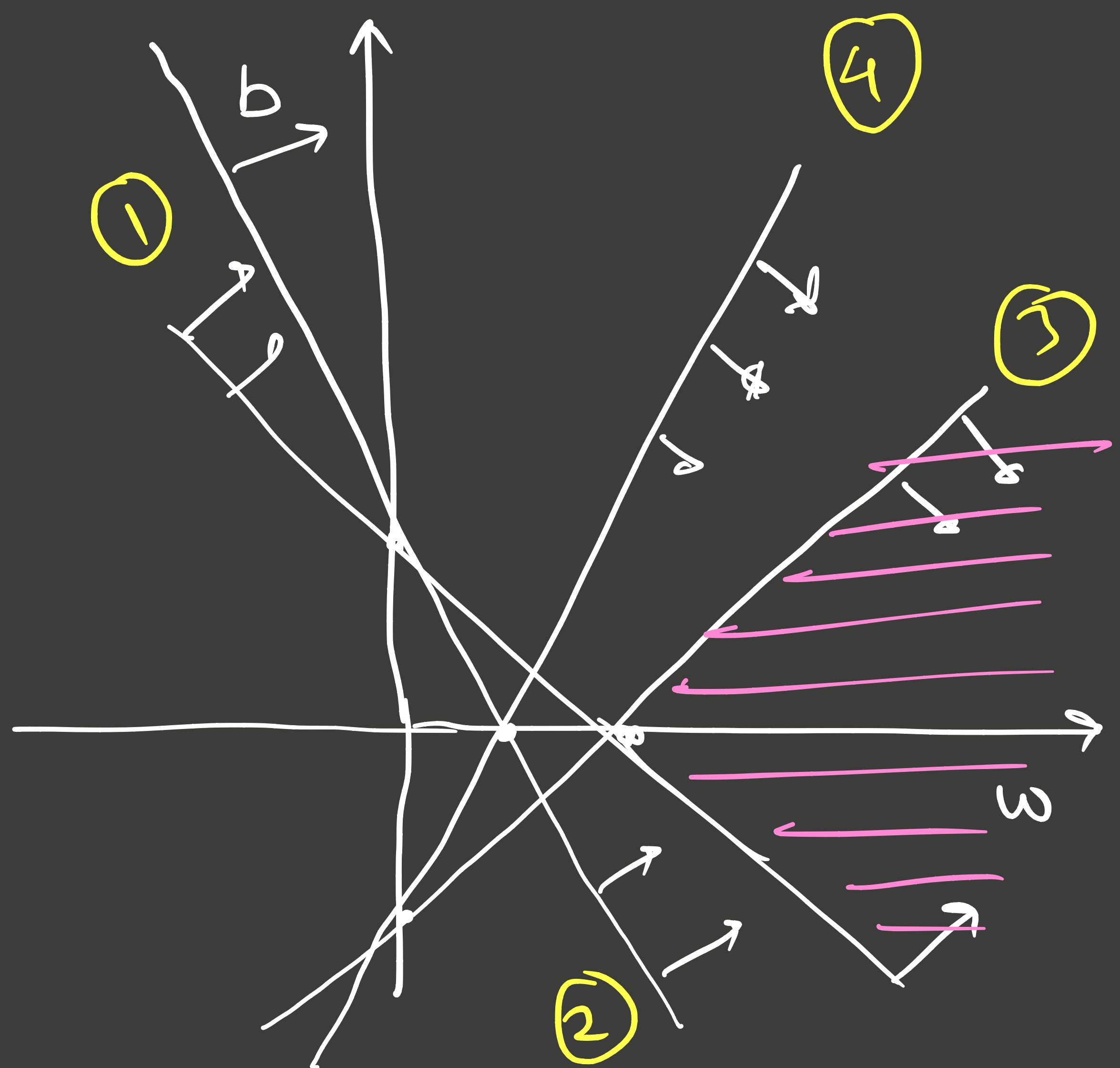
$$-1 (-2 w_1 + b) \geq 1 \quad \ldots (4)$$

$1(w_1 + b) \geq 1 \quad \cdots (1)$

$1(2w_1 + b) \geq 1 \quad \cdots (2)$

$-1(-w_1 + b) \geq 1 \quad \cdots (3) \Rightarrow w_1 - b \geq 1$

$-1(-2w_1 + b) \geq 1 \quad \cdots (4) \Rightarrow 2w_1 - b \geq 1$



$w_{min} = 1$

$b = 0$

$\vec{w} \cdot \bar{x} + b = 0$

or $x = 0$