

Decision Trees

Nipun Batra
Jan 8, 2019

Training Data

Day	Outlook	Temp	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	High	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Training Data

Day	Outlook	Temp	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Input features		Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	High	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Training Data

Day	Outlook	Temp	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Output Variable
D6	Rain			Strong	
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	High	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Example borrowed from Tom Mitchell's text book

Training Data

Discrete Output : Classification

Day	Outlook	Temp	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Output Variable
D6	Rain	Input features		Strong	
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	High	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Example borrowed from Tom Mitchell's text book

Training Data

Home #	Square footage	Age	Price (1000 USD)
D1	1000	20	30
D2	2000	10	No
D3	3000	5	40
D4	1240	10	50

Training Data

Home #	Square footage	Age	Price (1000 USD)
D1	1000	20	30
D2	200	10	No
D3	3000	5	40
D4	1240	10	50

A blue box labeled "Input" is positioned over the "Square footage" cell for D2.

Training Data

Home #	Square footage	Age	Price (1000 USD)
D1	1000	20	30
D2	200	10	40
D3	3000	5	50
D4	1240	10	30

The table illustrates training data for a model. The input features are Square footage and Age, and the output is Price (1000 USD). A blue box labeled "Input" highlights the Square footage and Age columns, and a blue box labeled "Output" highlights the Price column.

Training Data

Continuous Output : Regression

Home #	Square footage	Age	Price (1000 USD)
D1	1000	20	30
D2	200	10	40
D3	3000	5	50
D4	1240	10	30

The table illustrates training data for a regression model. The input features are Square footage and Age, and the output is Price (1000 USD). A blue box labeled "Input" highlights the Square footage and Age columns, and a blue box labeled "Output" highlights the Price column.

Train, Validation, Test

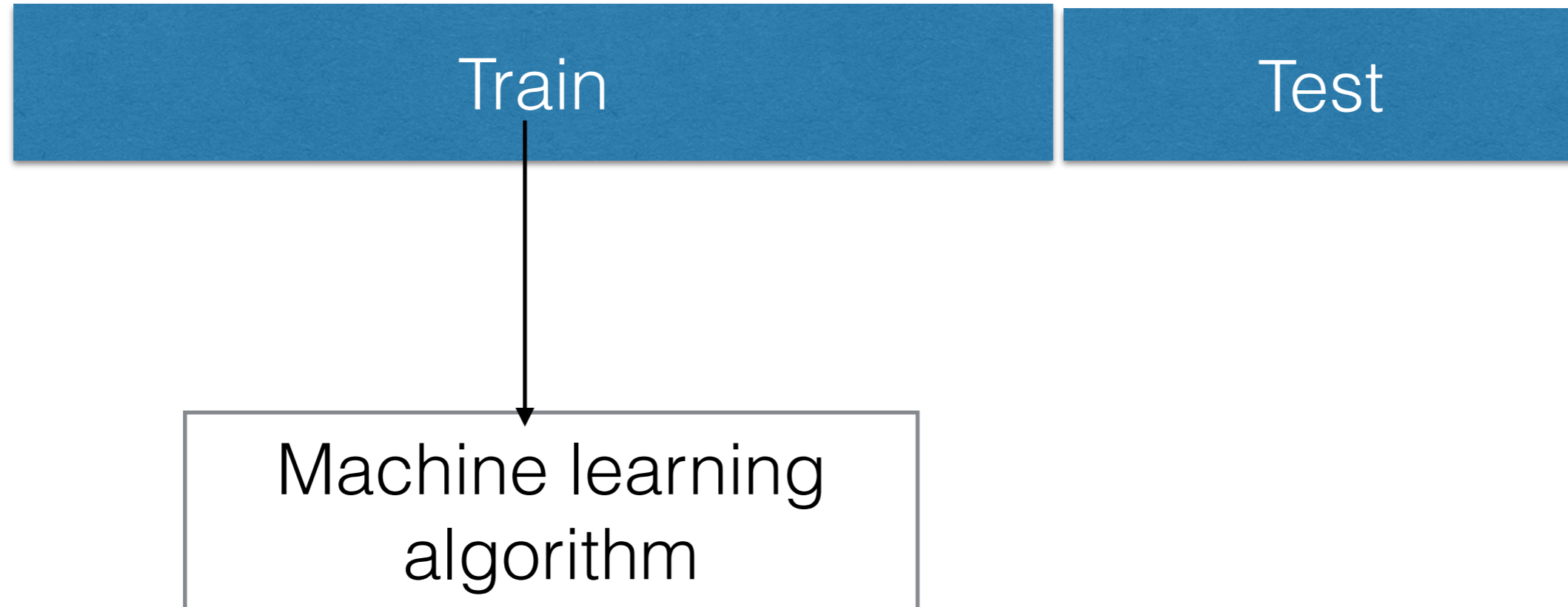
Data

Train, Validation, Test

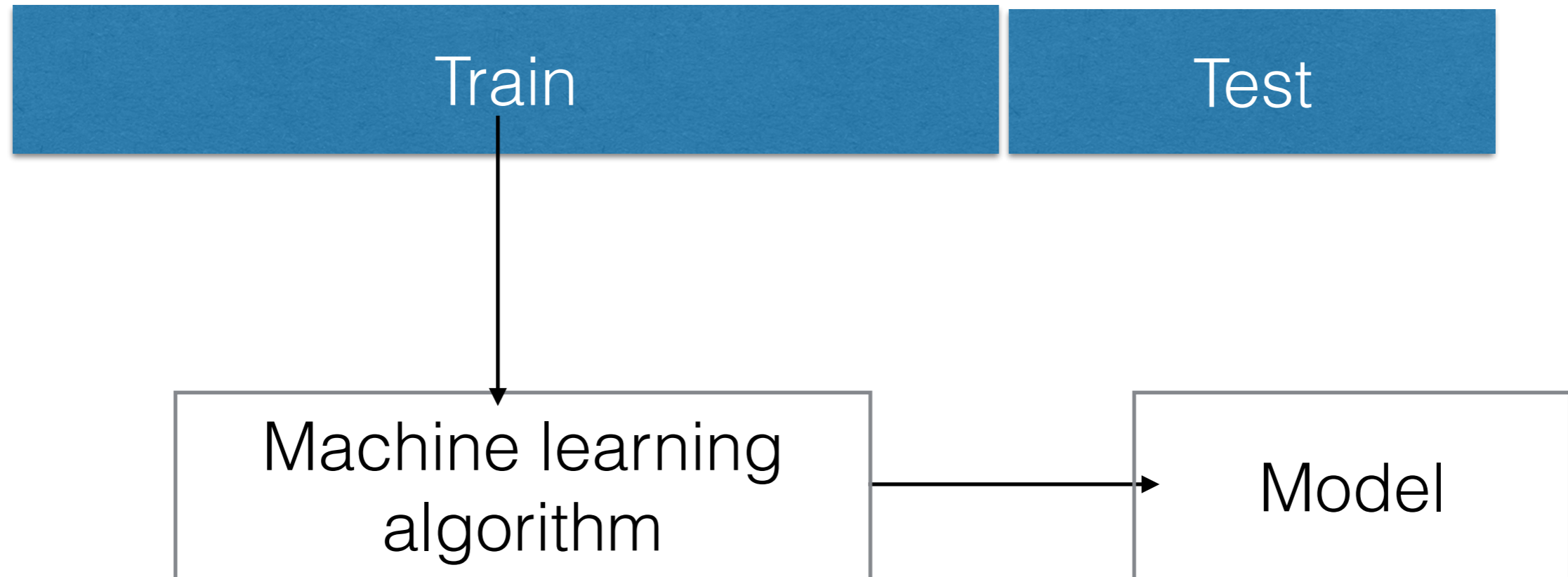
Train

Test

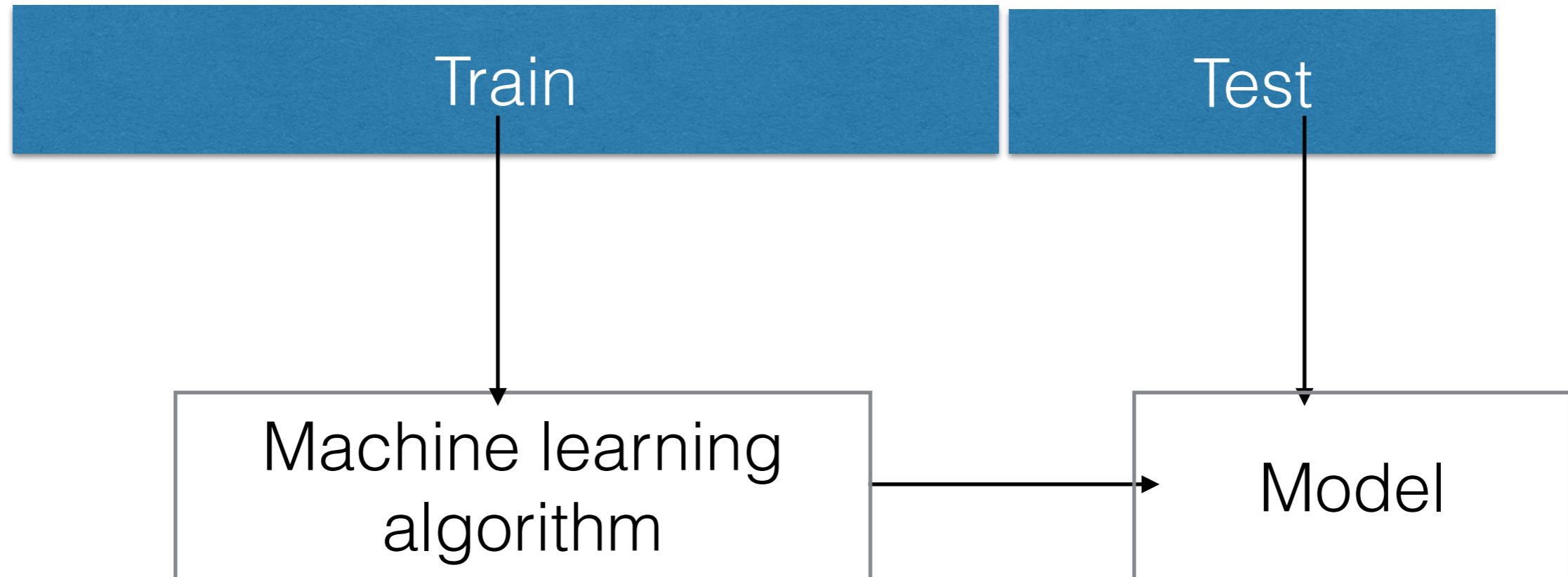
Train, Validation, Test



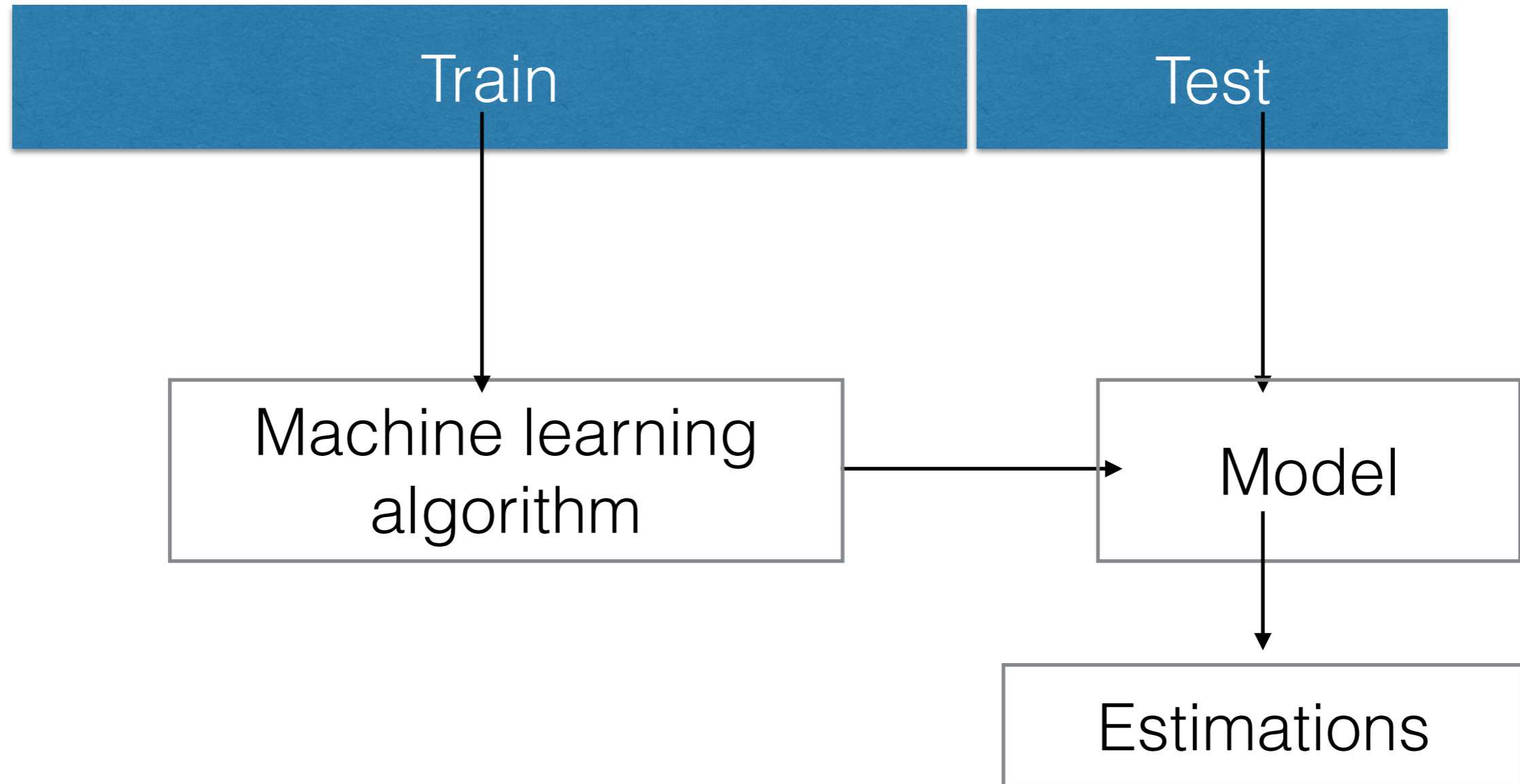
Train, Validation, Test



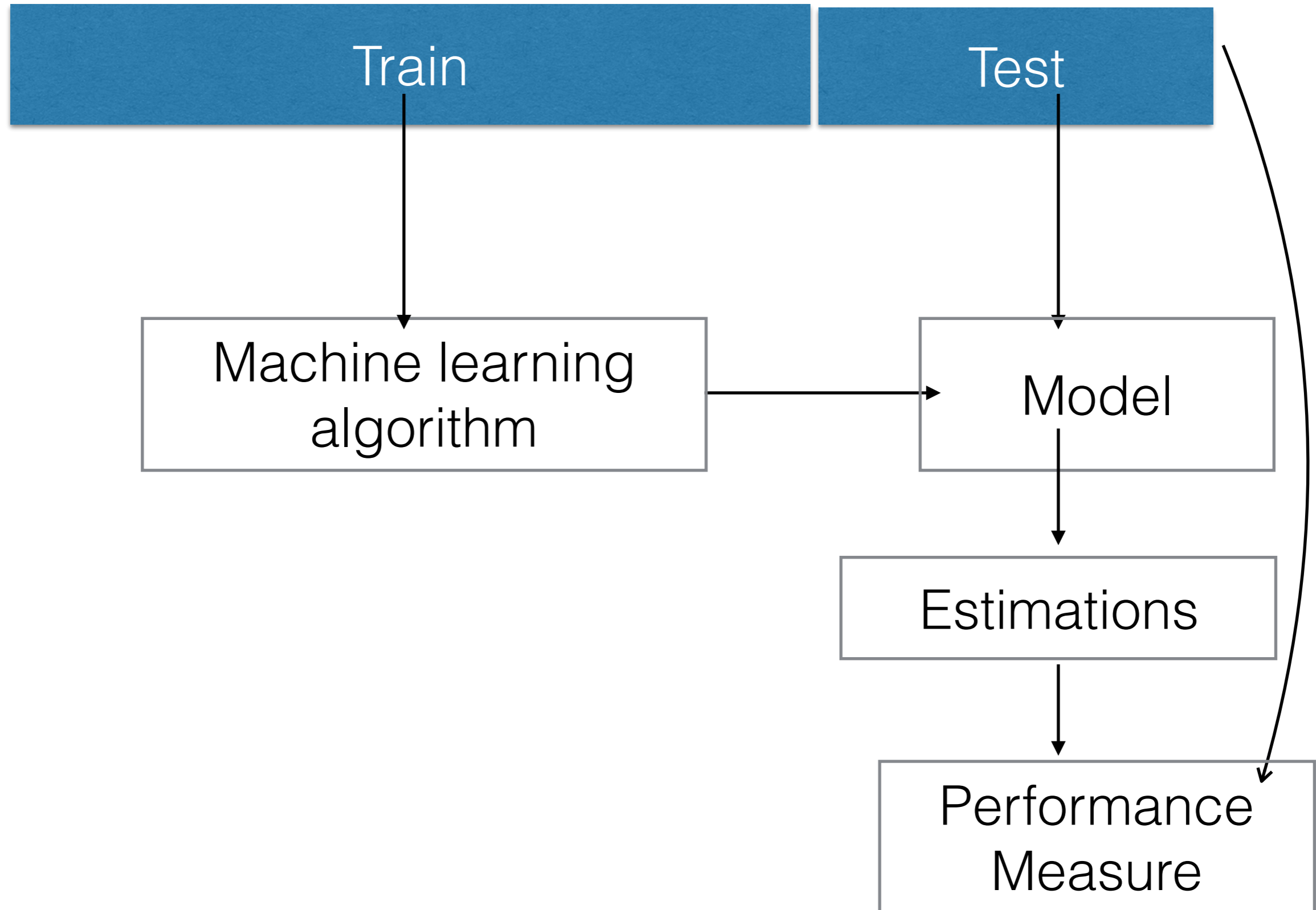
Train, Validation, Test



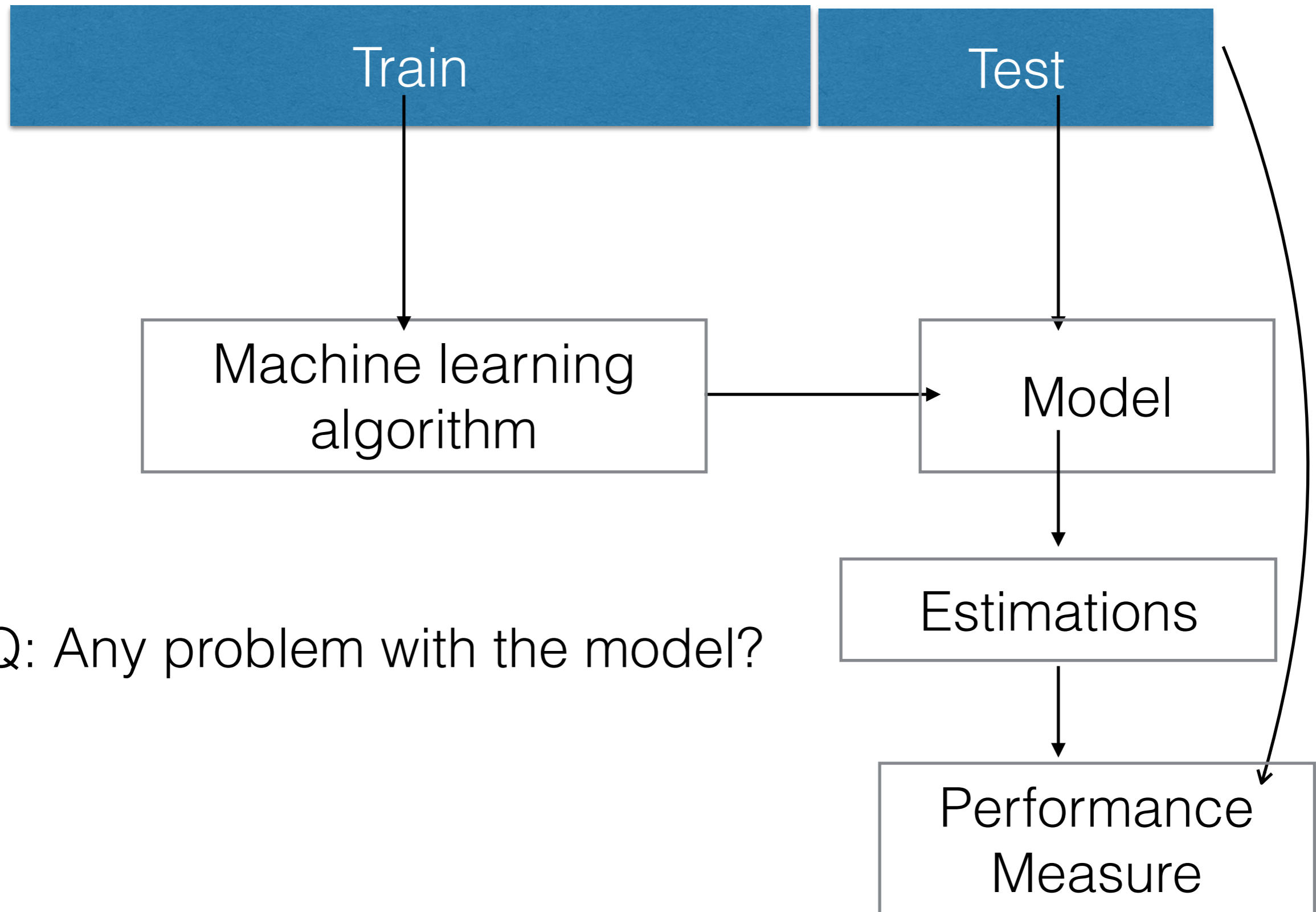
Train, Validation, Test



Train, Validation, Test

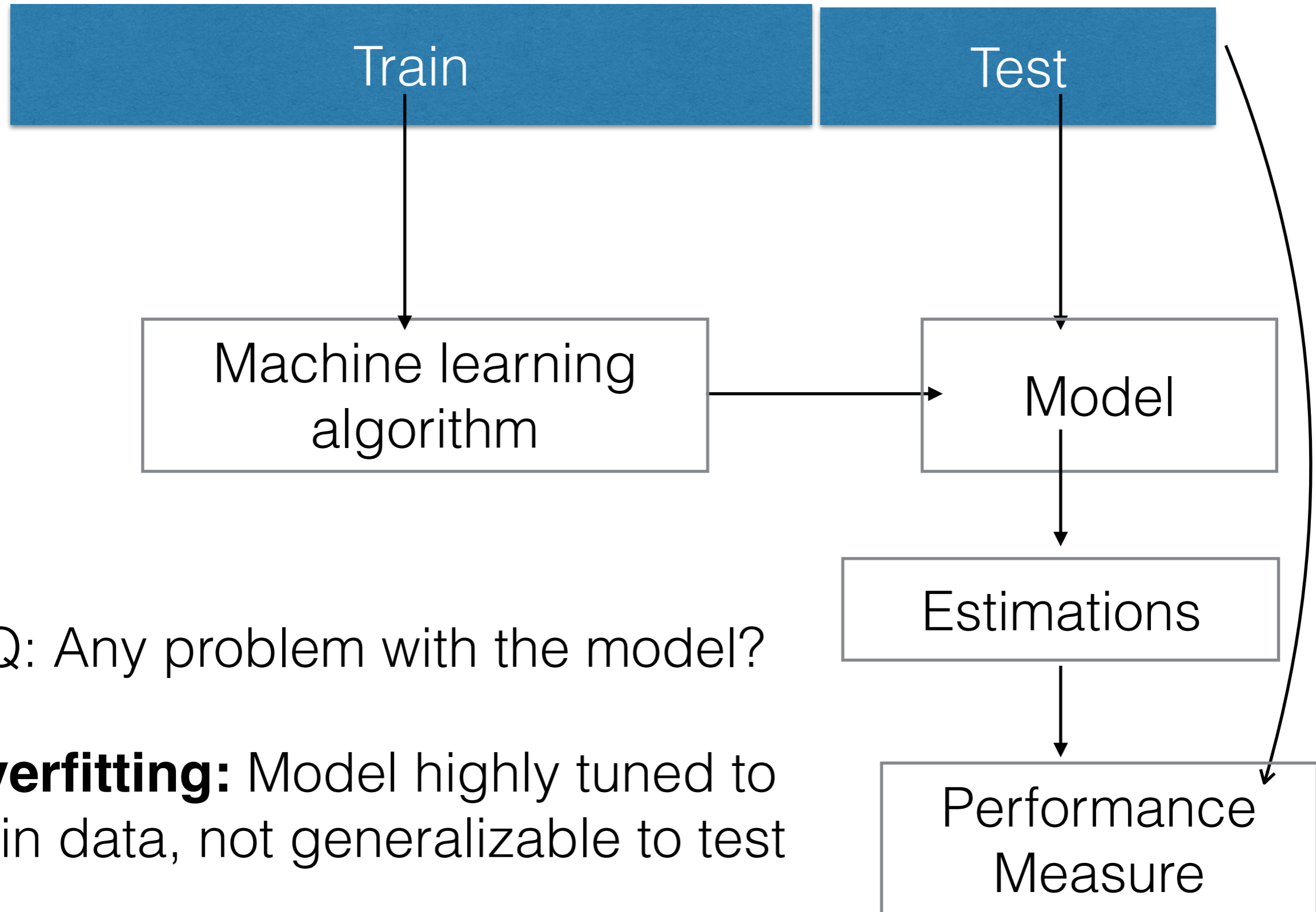


Train, Validation, Test



Q: Any problem with the model?

Train, Validation, Test



Q: Any problem with the model?

Overfitting: Model highly tuned to train data, not generalizable to test

Train, Validation, Test

Train

Test

Basic idea: Choose a model (parameters) which optimizes performance on validation set

Train, Validation, Test



Basic idea: Choose a model (parameters) which optimizes performance on validation set

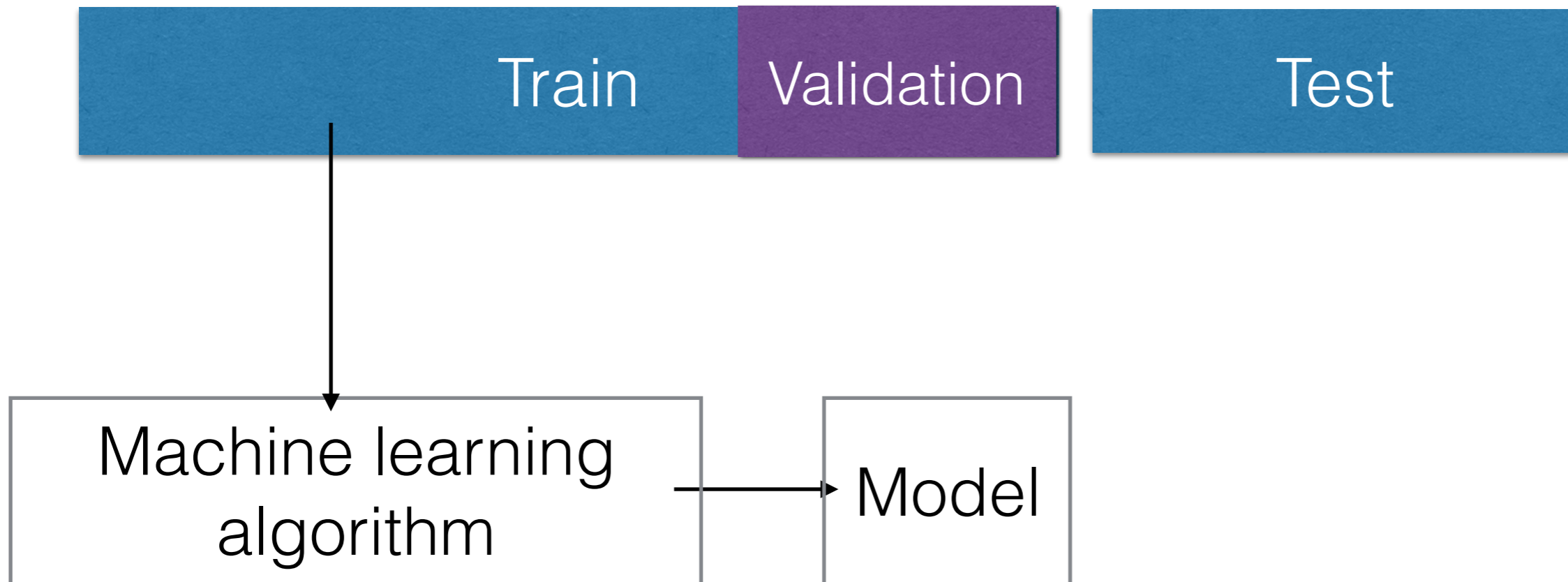
Train, Validation, Test



Machine learning
algorithm

Basic idea: Choose a model (parameters) which optimizes performance on validation set

Train, Validation, Test



Basic idea: Choose a model (parameters) which optimizes performance on validation set

Need For Interpretability

How to Maintain Trust in AI

Beyond developing initial trust, however, creators of AI also must work to maintain that trust. Siau and Wang suggest seven ways of “developing continuous trust” beyond the initial phases of product development:

- **Usability and reliability.** AI “should be designed to operate easily and intuitively,” Siau and Wang write. “There should be no unexpected downtime or crashes.”
- **Collaboration and communication.** AI developers want to create systems that perform autonomously, without human involvement. Developers must focus on creating AI applications that smoothly and easily collaborate and communicate with humans.
- **Sociability and bonding.** Building social activities into AI applications is one way to strengthen trust. A **robotic** dog that can recognize its owner and show affection is one example, Siau and Wang write.
- **Security and privacy protection.** AI applications rely on large data sets, so ensuring privacy and **security** will be crucial to establishing trust in the applications.
- **Interpretability.** Just as transparency is instrumental in **building** initial trust, interpretability – or the ability for a machine to explain its conclusions or actions – will help sustain trust.

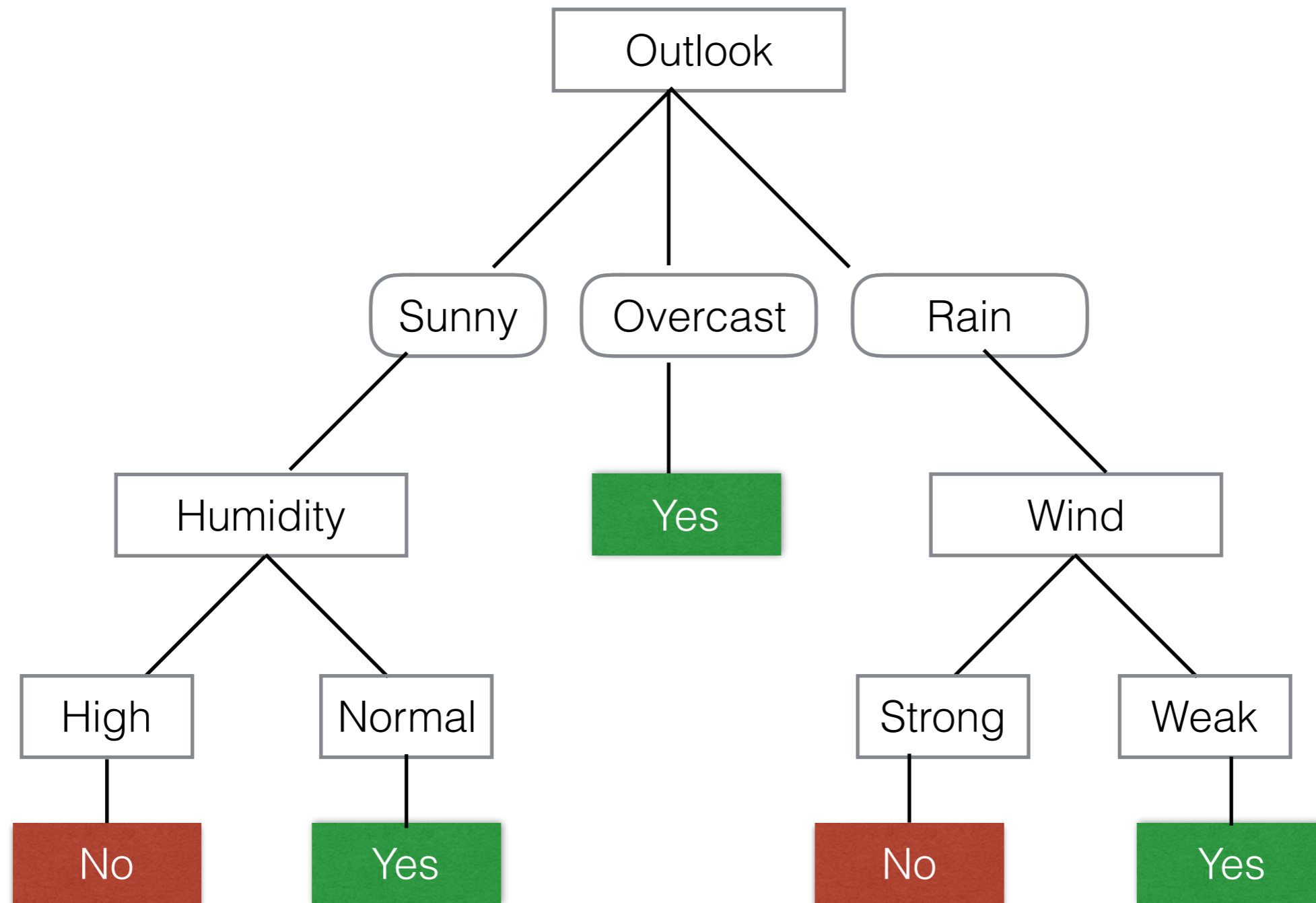
Training Data

Day	Outlook	Temp	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	High	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
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Training Data

Day	Outlook	Temp	Humidity	Wind	PlayTennis
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D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	High	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Decisions - Will I Play Tennis?



Hard to Learn Optimal Decision Tree

Volume 5, number 1

INFORMATION PROCESSING LETTERS

May 1976

CONSTRUCTING OPTIMAL BINARY DECISION TREES IS NP-COMPLETE*

Laurent HYAFIL

IRIA – Laboria, 78150 Rocquencourt, France

and

Ronald L. RIVEST

Dept. of Electrical Engineering and Computer Science, M.I.T., Cambridge, Massachusetts 02139, USA

Received 7 November 1975, revised version received 26 January 1976

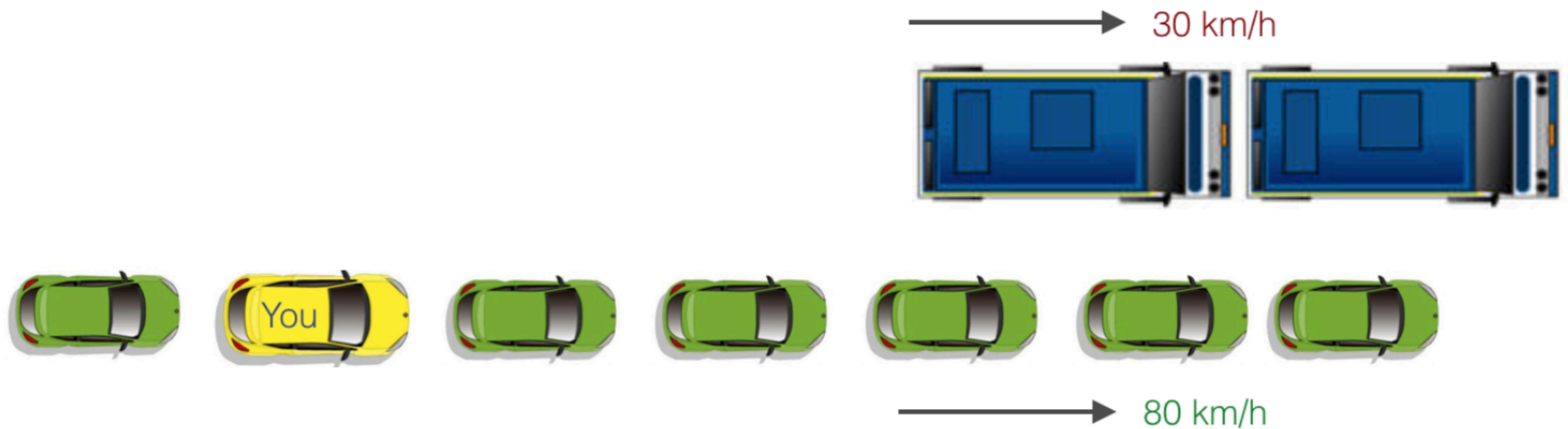
Binary decision trees, computational complexity, NP-complete

Greedy Algorithm

Intuition: At each level, choose an attribute that gives “biggest estimated performance gain”

Greedy Algorithm

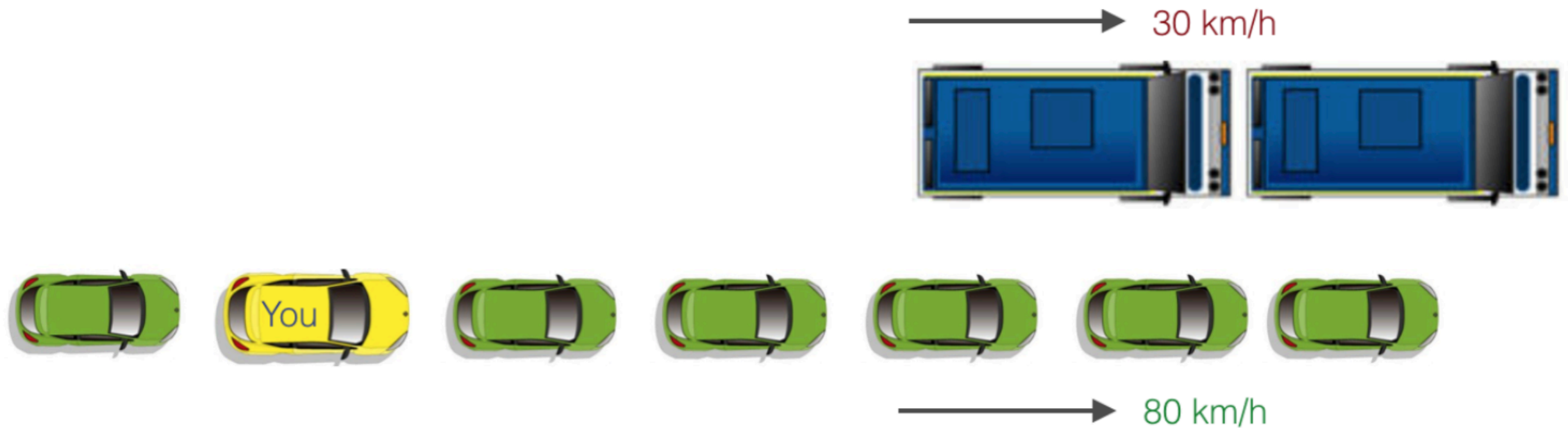
Intuition: At each level, choose an attribute that gives “biggest estimated performance gain”



Greedy Algorithm

Intuition: At each level, choose an attribute that gives “biggest estimated performance gain”

Greedy! = Optimal



Greedy Algorithm

ID3 (Examples, Target_Attribute, Attributes)

Greedy Algorithm

ID3 (Examples, Target_Attribute, Attributes)

1. Create a root node for tree

Greedy Algorithm

ID3 (Examples, Target_Attribute, Attributes)

1. Create a root node for tree
2. If all examples are +/-, return root with label = +/-

Greedy Algorithm

ID3 (Examples, Target_Attribute, Attributes)

1. Create a root node for tree
2. If all examples are +/-, return root with label = +/-
3. If attributes = empty, return root with most common value of Target_Attribute in Examples

Greedy Algorithm

ID3 (Examples, Target_Attribute, Attributes)

1. Create a root node for tree
2. If all examples are +/-, return root with label = +/-
3. If attributes = empty, return root with most common value of Target_Attribute in Examples
4. Begin

Greedy Algorithm

ID3 (Examples, Target_Attribute, Attributes)

1. Create a root node for tree
2. If all examples are +/-, return root with label = +/-
3. If attributes = empty, return root with most common value of Target_Attribute in Examples
4. Begin
 1. $A \leftarrow$ attribute from Attributes which **best** classifies Examples

Greedy Algorithm

ID3 (Examples, Target_Attribute, Attributes)

1. Create a root node for tree
2. If all examples are +/-, return root with label = +/-
3. If attributes = empty, return root with most common value of Target_Attribute in Examples
4. Begin
 1. $A \leftarrow$ attribute from Attributes which **best** classifies Examples
 2. $Root \leftarrow A$

Greedy Algorithm

ID3 (Examples, Target_Attribute, Attributes)

1. Create a root node for tree
2. If all examples are +/-, return root with label = +/-
3. If attributes = empty, return root with most common value of Target_Attribute in Examples
4. Begin
 1. $A \leftarrow$ attribute from Attributes which **best** classifies Examples
 2. Root \leftarrow A
 3. For each value (v) of A

Greedy Algorithm

ID3 (Examples, Target_Attribute, Attributes)

1. Create a root node for tree
2. If all examples are +/-, return root with label = +/-
3. If attributes = empty, return root with most common value of Target_Attribute in Examples
4. Begin
 1. $A \leftarrow$ attribute from Attributes which **best** classifies Examples
 2. Root \leftarrow A
 3. For each value (v) of A
 1. Add new tree branch : $A = v$

Greedy Algorithm

ID3 (Examples, Target_Attribute, Attributes)

1. Create a root node for tree
2. If all examples are +/-, return root with label = +/-
3. If attributes = empty, return root with most common value of Target_Attribute in Examples
4. Begin
 1. $A \leftarrow$ attribute from Attributes which **best** classifies Examples
 2. Root \leftarrow A
 3. For each value (v) of A
 1. Add new tree branch : $A = v$
 2. Examples_v : subset of examples that $A = v$

Greedy Algorithm

ID3 (Examples, Target_Attribute, Attributes)

1. Create a root node for tree
2. If all examples are +/-, return root with label = +/-
3. If attributes = empty, return root with most common value of Target_Attribute in Examples
4. Begin
 1. $A \leftarrow$ attribute from Attributes which **best** classifies Examples
 2. Root \leftarrow A
 3. For each value (v) of A
 1. Add new tree branch : $A = v$
 2. Examples_v : subset of examples that $A = v$
 3. If Examples_v is empty: add leaf with label = most common value of Target_Attribute

Greedy Algorithm

ID3 (Examples, Target_Attribute, Attributes)

1. Create a root node for tree
2. If all examples are +/-, return root with label = +/-
3. If attributes = empty, return root with most common value of Target_Attribute in Examples
4. Begin
 1. $A \leftarrow$ attribute from Attributes which **best** classifies Examples
 2. Root $\leftarrow A$
 3. For each value (v) of A
 1. Add new tree branch : $A = v$
 2. Examples_ v : subset of examples that $A = v$
 3. If Examples_ v is empty: add leaf with label = most common value of Target_Attribute
 4. Else: ID3 (Examples_ v , Target_attribute, Attributes - { A })

Entropy

Entropy: Statistical measure to characterize the (im)purity of examples

Entropy

Entropy: Statistical measure to characterize the (im)purity of examples

PlayTennis
No
No
Yes
Yes
Yes
No
Yes
No
Yes
Yes
Yes
Yes
Yes
No

Entropy

Entropy: Statistical measure to characterize the (im)purity of examples

PlayTennis
No
No
Yes
Yes
Yes
No
Yes
No
Yes
Yes
Yes
Yes
Yes
No

5 No, 9 Yes

Entropy

Entropy: Statistical measure to characterize the (im)purity of examples

PlayTennis
No
No
Yes
Yes
Yes
No
Yes
No
Yes
Yes
Yes
Yes
Yes
No

Entropy

Entropy: Statistical measure to characterize the (im)purity of examples

Entropy

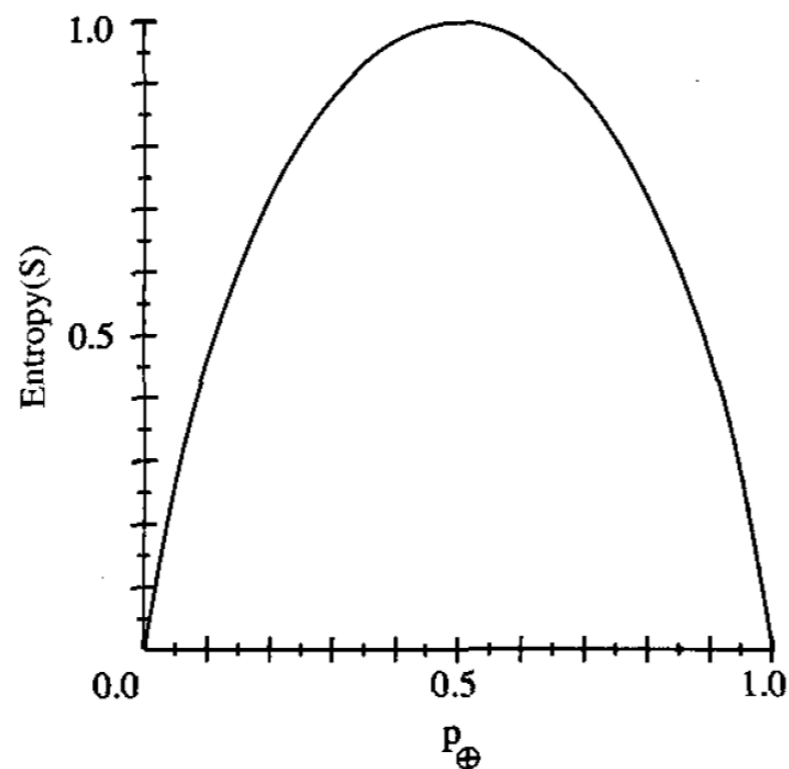
Entropy: Statistical measure to characterize the (im)purity of examples

$$\begin{aligned}\text{Entropy} &= - p_{\text{No}} \log_2 p_{\text{No}} - p_{\text{Yes}} \log_2 p_{\text{Yes}} \\ &= -(5/14) \log_2(5/14) - (9/14) \log_2(9/14) \\ &= 0.94\end{aligned}$$

Entropy

Entropy: Statistical measure to characterize the (im)purity of examples

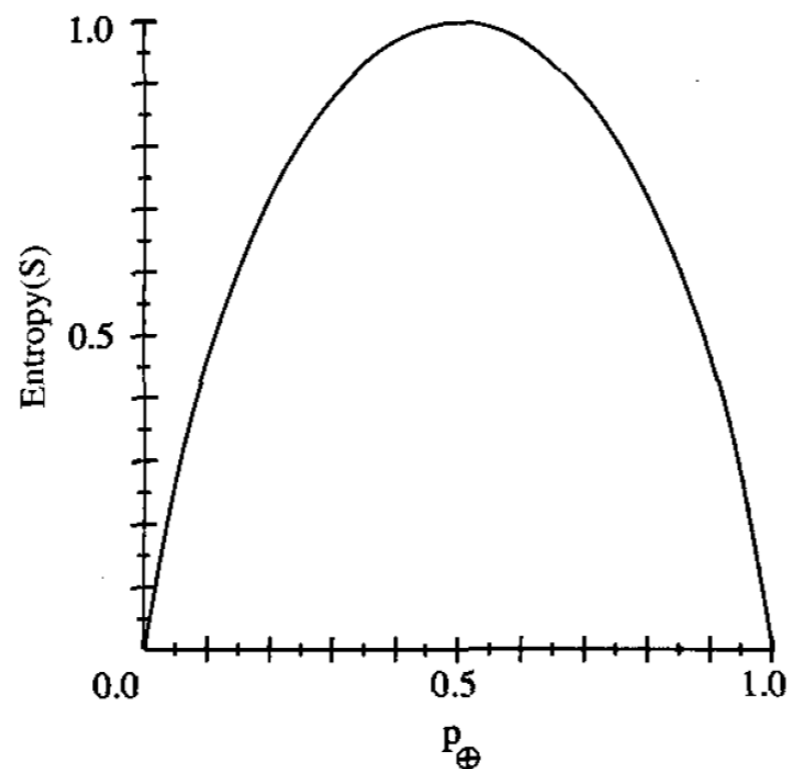
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Entropy

Entropy: Statistical measure to characterize the (im)purity of examples

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Avg. # of bits to transmit

Information Gain

Information Gain: Reduction in entropy

Information Gain

Information Gain: Reduction in entropy

By partitioning examples (S) on attribute A

Information Gain

By partitioning examples (S) on attribute A

Information Gain

Information Gain

$$\mathit{Gain}(S, A) \equiv \mathit{Entropy}(S) - \sum_{v \in \mathit{Values}(A)} \frac{|S_v|}{|S|} \mathit{Entropy}(S_v)$$

Information Gain

$$Gain(S, A) \equiv Entropy(S) - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

Wind	PlayTennis
Weak	No
Strong	No
Weak	Yes
Weak	Yes
Weak	Yes
Strong	No
Strong	Yes
Weak	No
Weak	Yes
Weak	Yes
Strong	Yes
Strong	Yes
Weak	Yes
Strong	No

Information Gain

$$Gain(S, A) \equiv Entropy(S) - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

Wind	PlayTennis
Weak	No
Strong	No
Weak	Yes
Weak	Yes
Weak	Yes
Strong	No
Strong	Yes
Weak	No
Weak	Yes
Weak	Yes
Strong	Yes
Strong	Yes
Weak	Yes
Strong	No

- A = Wind

Information Gain

$$Gain(S, A) \equiv Entropy(S) - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

Wind	PlayTennis
Weak	No
Strong	No
Weak	Yes
Weak	Yes
Weak	Yes
Strong	No
Strong	Yes
Weak	No
Weak	Yes
Weak	Yes
Strong	Yes
Strong	Yes
Weak	Yes
Strong	No

- A = Wind
- Values (Wind) = Weak, Strong

Information Gain

$$Gain(S, A) \equiv Entropy(S) - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

Wind	PlayTennis
Weak	No
Strong	No
Weak	Yes
Weak	Yes
Weak	Yes
Strong	No
Strong	Yes
Weak	No
Weak	Yes
Weak	Yes
Strong	Yes
Strong	Yes
Weak	Yes
Strong	No

- A = Wind
- Values (Wind) = Weak, Strong
- S = [9+, 5-]

Information Gain

$$Gain(S, A) \equiv Entropy(S) - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

Wind	PlayTennis
Weak	No
Strong	No
Weak	Yes
Weak	Yes
Weak	Yes
Strong	No
Strong	Yes
Weak	No
Weak	Yes
Weak	Yes
Strong	Yes
Strong	Yes
Weak	Yes
Strong	No

- $A = \text{Wind}$
- $Values(\text{Wind}) = \text{Weak, Strong}$
- $S = [9+, 5-]$
- $S_{\text{Weak}} = [6+, 2-]$

Information Gain

$$Gain(S, A) \equiv Entropy(S) - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

Wind	PlayTennis
Weak	No
Strong	No
Weak	Yes
Weak	Yes
Weak	Yes
Strong	No
Strong	Yes
Weak	No
Weak	Yes
Weak	Yes
Strong	Yes
Strong	Yes
Weak	Yes
Strong	No

- A = Wind
- Values (Wind) = Weak, Strong
- S = [9+, 5-]
- S_{Weak} = [6+, 2-]
- S_{Strong} = [3+, 3-]

Information Gain

$$Gain(S, A) \equiv Entropy(S) - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

Wind	PlayTennis
Weak	No
Strong	No
Weak	Yes
Weak	Yes
Weak	Yes
Strong	No
Strong	Yes
Weak	No
Weak	Yes
Weak	Yes
Strong	Yes
Strong	Yes
Weak	Yes
Strong	No

- A = Wind
- Values (Wind) = Weak, Strong
- S = [9+, 5-]
- S_{Weak} = [6+, 2-]
- S_{Strong} = [3+, 3-]
- Gain (S, Wind) = Entropy (S) - (8/14)*Entropy (S_{Weak}) - (6/14)*Entropy(S_{Strong})

Information Gain

$$Gain(S, A) \equiv Entropy(S) - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

Wind	PlayTennis
Weak	No
Strong	No
Weak	Yes
Weak	Yes
Weak	Yes
Strong	No
Strong	Yes
Weak	No
Weak	Yes
Weak	Yes
Strong	Yes
Strong	Yes
Weak	Yes
Strong	No

- A = Wind
- Values (Wind) = Weak, Strong
- S = [9+, 5-]
- S_{Weak} = [6+, 2-]
- S_{Strong} = [3+, 3-]
- Gain (S, Wind) = Entropy (S) - (8/14)*Entropy (S_{Weak}) - (6/14)*Entropy(S_{Strong}) = 0.048

Information Gain

Outlook	PlayTennis
Sunny	No
Sunny	No
Overcast	Yes
Rain	Yes
Rain	Yes
Rain	No
Overcast	Yes
Sunny	No
Sunny	Yes
Rain	Yes
Sunny	Yes
Overcast	Yes
Overcast	Yes
Rain	No

Information Gain

Outlook	PlayTennis
Sunny	No
Sunny	No
Overcast	Yes
Rain	Yes
Rain	Yes
Rain	No
Overcast	Yes
Sunny	No
Sunny	Yes
Rain	Yes
Sunny	Yes
Overcast	Yes
Overcast	Yes
Rain	No

- $A = \text{Outlook}$

Information Gain

Outlook	PlayTennis
Sunny	No
Sunny	No
Overcast	Yes
Rain	Yes
Rain	Yes
Rain	No
Overcast	Yes
Sunny	No
Sunny	Yes
Rain	Yes
Sunny	Yes
Overcast	Yes
Overcast	Yes
Rain	No

- $A = \text{Outlook}$
- Values (Outlook) = Sunny, Overcast, Rain

Information Gain

Outlook	PlayTennis
Sunny	No
Sunny	No
Overcast	Yes
Rain	Yes
Rain	Yes
Rain	No
Overcast	Yes
Sunny	No
Sunny	Yes
Rain	Yes
Sunny	Yes
Overcast	Yes
Overcast	Yes
Rain	No

- $A = \text{Outlook}$
- Values (Outlook) = Sunny, Overcast, Rain
- $S = [9+, 5-]$

Information Gain

Outlook	PlayTennis
Sunny	No
Sunny	No
Overcast	Yes
Rain	Yes
Rain	Yes
Rain	No
Overcast	Yes
Sunny	No
Sunny	Yes
Rain	Yes
Sunny	Yes
Overcast	Yes
Overcast	Yes
Rain	No

- $A = \text{Outlook}$
- Values (Outlook) = Sunny, Overcast, Rain
- $S = [9+, 5-]$
- $S_{\text{Sunny}} = [2+, 3-]$

Information Gain

Outlook	PlayTennis
Sunny	No
Sunny	No
Overcast	Yes
Rain	Yes
Rain	Yes
Rain	No
Overcast	Yes
Sunny	No
Sunny	Yes
Rain	Yes
Sunny	Yes
Overcast	Yes
Overcast	Yes
Rain	No

- $A = \text{Outlook}$
- Values (Outlook) = Sunny, Overcast, Rain
- $S = [9+, 5-]$
- $S_{\text{Sunny}} = [2+, 3-]$
- $S_{\text{Overcast}} = [4+, 0-]$

Information Gain

Outlook	PlayTennis
Sunny	No
Sunny	No
Overcast	Yes
Rain	Yes
Rain	Yes
Rain	No
Overcast	Yes
Sunny	No
Sunny	Yes
Rain	Yes
Sunny	Yes
Overcast	Yes
Overcast	Yes
Rain	No

- $A = \text{Outlook}$
- Values (Outlook) = Sunny, Overcast, Rain
- $S = [9+, 5-]$
- $S_{\text{Sunny}} = [2+, 3-]$
- $S_{\text{Overcast}} = [4+, 0-]$
- $S_{\text{Rain}} = [3+, 2-]$

Information Gain

Outlook	PlayTennis
Sunny	No
Sunny	No
Overcast	Yes
Rain	Yes
Rain	Yes
Rain	No
Overcast	Yes
Sunny	No
Sunny	Yes
Rain	Yes
Sunny	Yes
Overcast	Yes
Overcast	Yes
Rain	No

- $A = \text{Outlook}$
- Values (Outlook) = Sunny, Overcast, Rain
- $S = [9+, 5-]$
- $S_{\text{Sunny}} = [2+, 3-]$
- $S_{\text{Overcast}} = [4+, 0-]$
- $S_{\text{Rain}} = [3+, 2-]$
- $\text{Gain}(S, \text{Outlook}) = \text{Entropy}(S) - (5/14) * \text{Entropy}(S_{\text{Sunny}}) - (4/14) * \text{Entropy}(S_{\text{Overcast}}) -$

Information Gain

Outlook	PlayTennis
Sunny	No
Sunny	No
Overcast	Yes
Rain	Yes
Rain	Yes
Rain	No
Overcast	Yes
Sunny	No
Sunny	Yes
Rain	Yes
Sunny	Yes
Overcast	Yes
Overcast	Yes
Rain	No

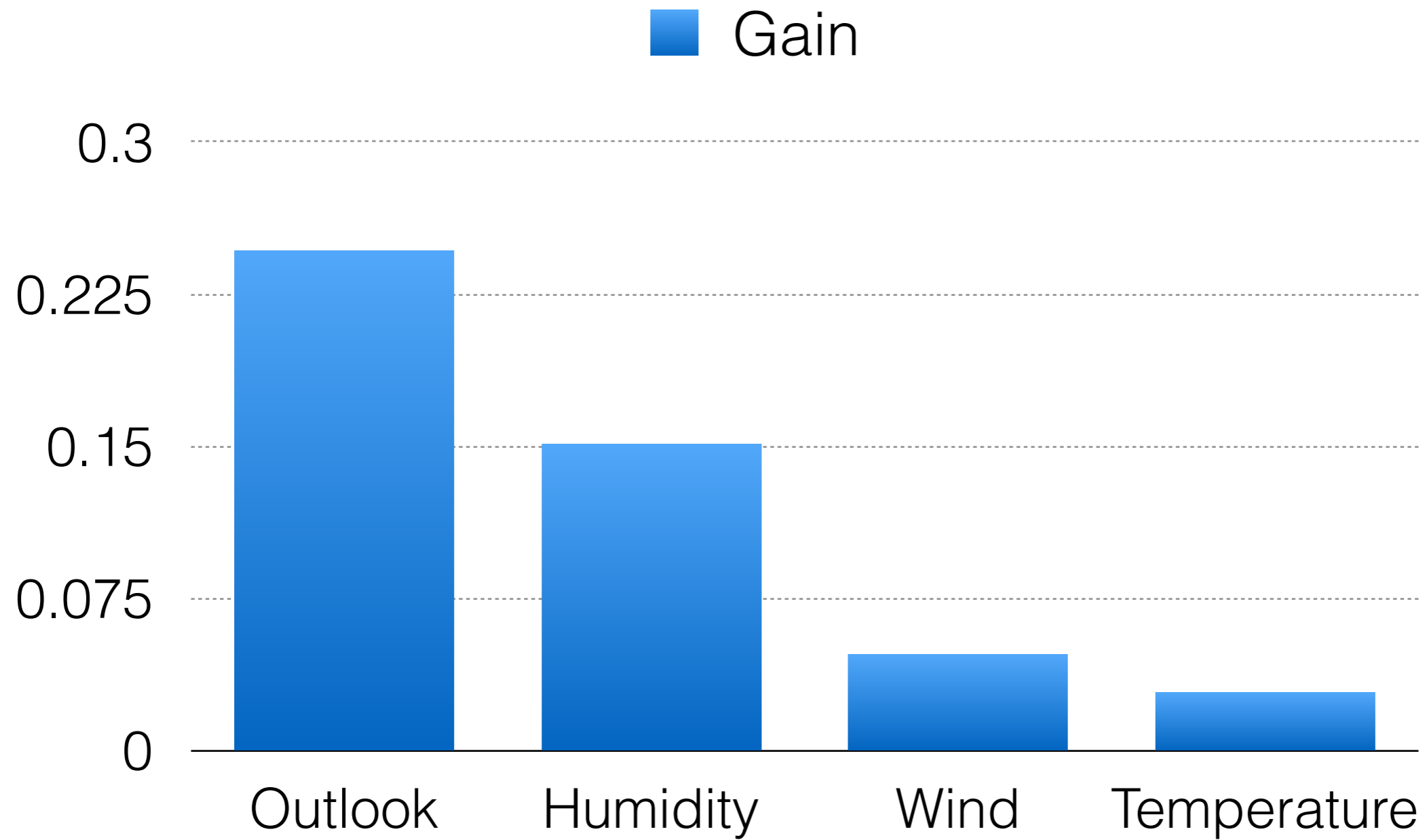
- $A = \text{Outlook}$
- Values (Outlook) = Sunny, Overcast, Rain
- $S = [9+, 5-]$
- $S_{\text{Sunny}} = [2+, 3-]$
- $S_{\text{Overcast}} = [4+, 0-]$
- $S_{\text{Rain}} = [3+, 2-]$
- $\text{Gain}(S, \text{Outlook}) = \text{Entropy}(S) - (5/14) * \text{Entropy}(S_{\text{Sunny}}) - (4/14) * \text{Entropy}(S_{\text{Overcast}}) - (5/14) * \text{Entropy}(S_{\text{Rain}})$

Information Gain

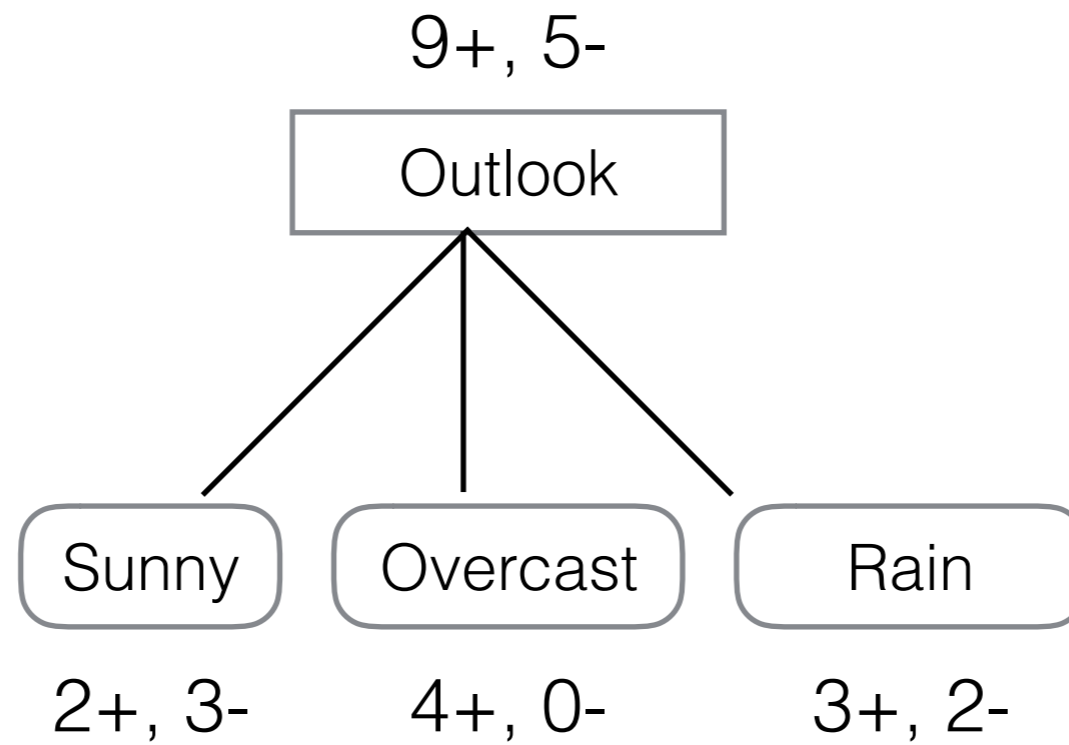
Outlook	PlayTennis
Sunny	No
Sunny	No
Overcast	Yes
Rain	Yes
Rain	Yes
Rain	No
Overcast	Yes
Sunny	No
Sunny	Yes
Rain	Yes
Sunny	Yes
Overcast	Yes
Overcast	Yes
Rain	No

- $A = \text{Outlook}$
- Values (Outlook) = Sunny, Overcast, Rain
- $S = [9+, 5-]$
- $S_{\text{Sunny}} = [2+, 3-]$
- $S_{\text{Overcast}} = [4+, 0-]$
- $S_{\text{Rain}} = [3+, 2-]$
- $\text{Gain}(S, \text{Outlook}) = \text{Entropy}(S) - (5/14) * \text{Entropy}(S_{\text{Sunny}}) - (4/14) * \text{Entropy}(S_{\text{Overcast}}) - (5/14) * \text{Entropy}(S_{\text{Rain}})$
 $= 0.246$

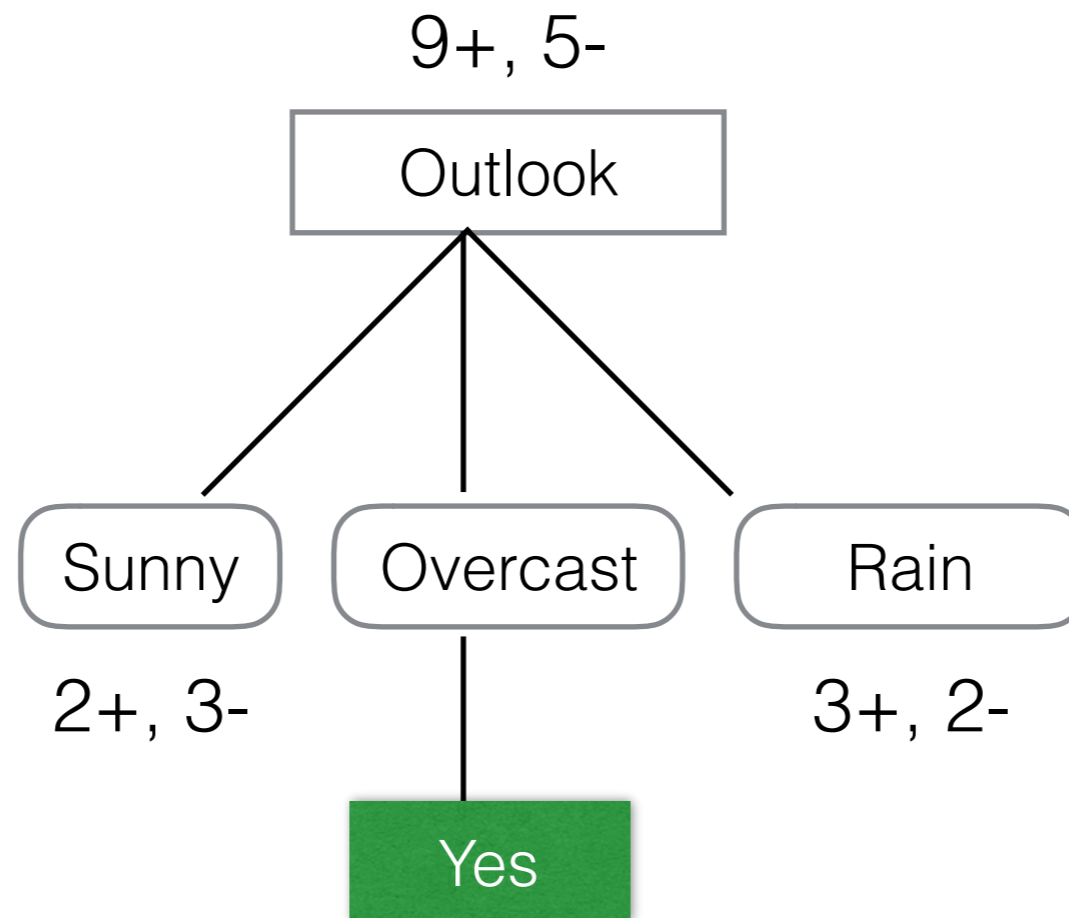
Information Gain



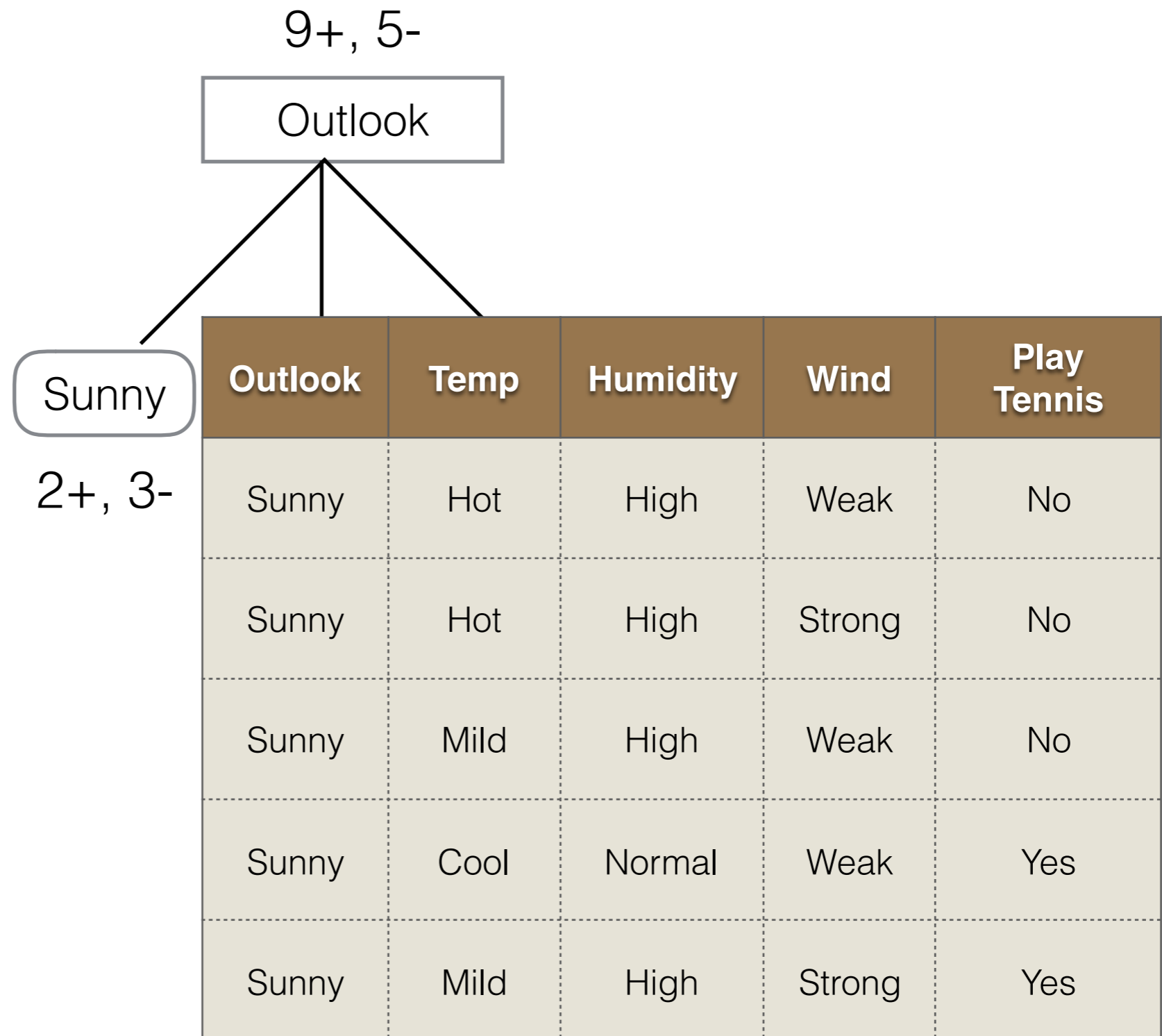
Worked Out Example



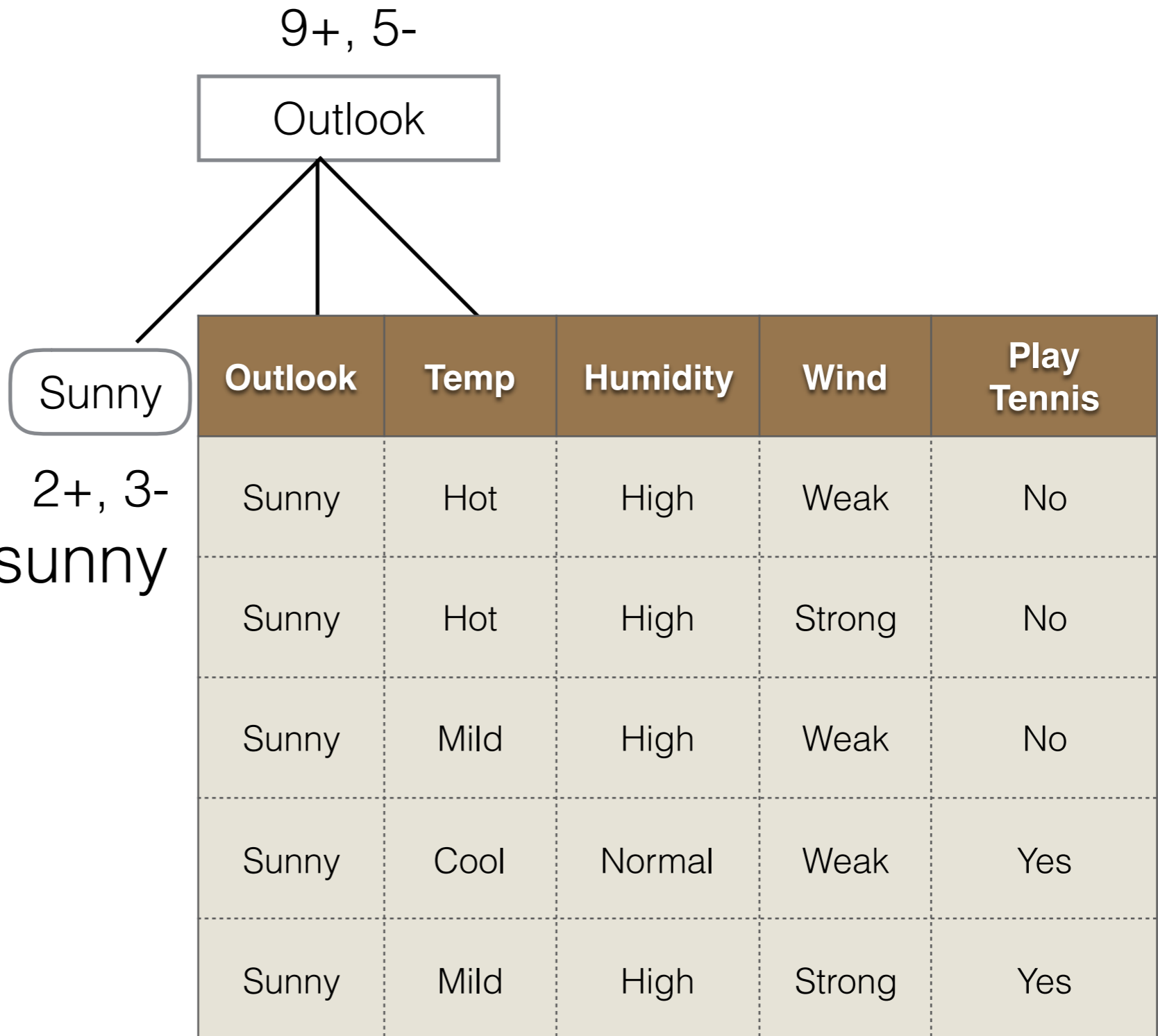
Worked Out Example



Worked Out Example



Worked Out Example



2+, 3-

S' = S_outlook is sunny

Entropy (S') =

$$-(2/5) \log_2(2/5) - (3/5) \log_2(3/5)$$

Worked Out Example

