Q) Why minimize (Primal) $\Leftrightarrow$ Maximize (Dual)?

Minimize $||\vec{w}||^2$ $\Rightarrow$ Maximize $L(\alpha)$

s.t.

$y_i(\bar{w}.x_i+b) \geq 1$

s.t.

$$\sum_{i=1}^{N} \alpha_i y_i = 0$$

$$\alpha_i \geq 0 \quad \forall i \in \{1..N\}$$

Q) Minimize x^2

s.t.

$x \leq 2$ or $x - 2 \leq 0$

$$J(x) = \begin{cases} x^2 & : x \leq 2 \\ \infty & o/w \end{cases}$$

$$= x^2 + I_\infty[x-2]$$

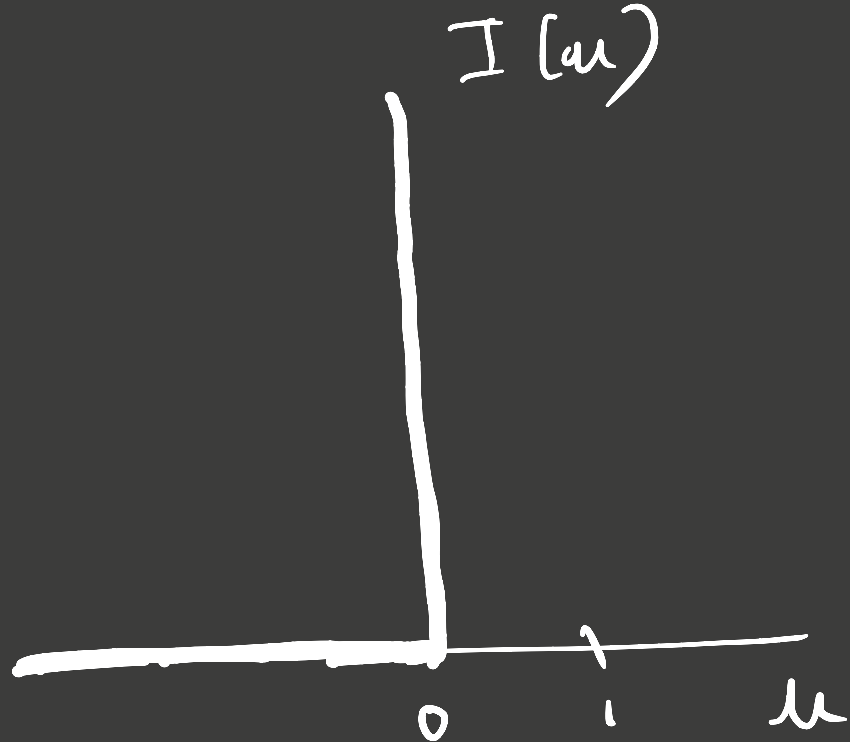
Objective: Minimize $J(x)$

$$I_\infty[u] = \begin{cases} 0 & : u \leq 0 \\ \infty & : o/w \end{cases}$$

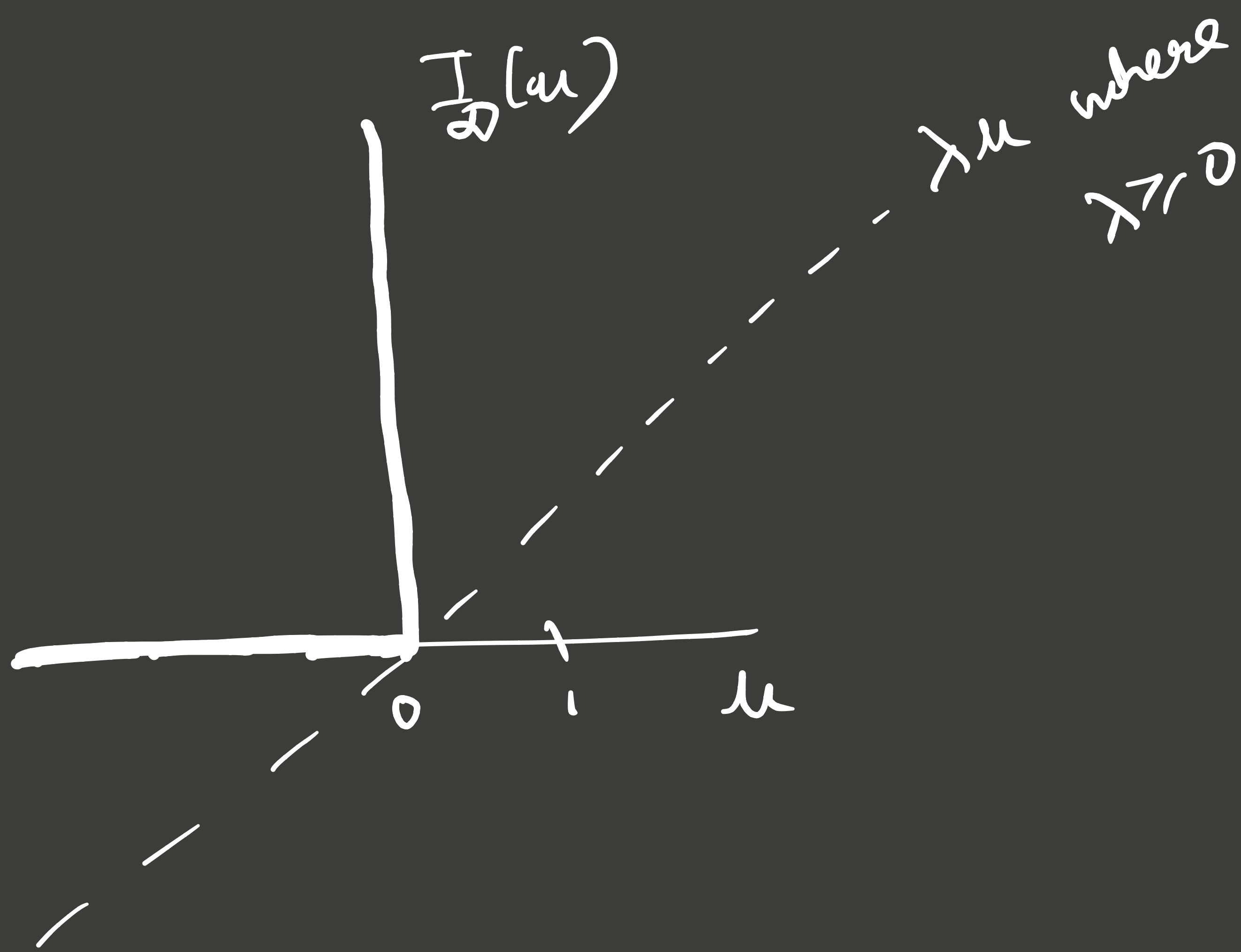
$I(u)$ is

× non-differentiable

× dis continuous.



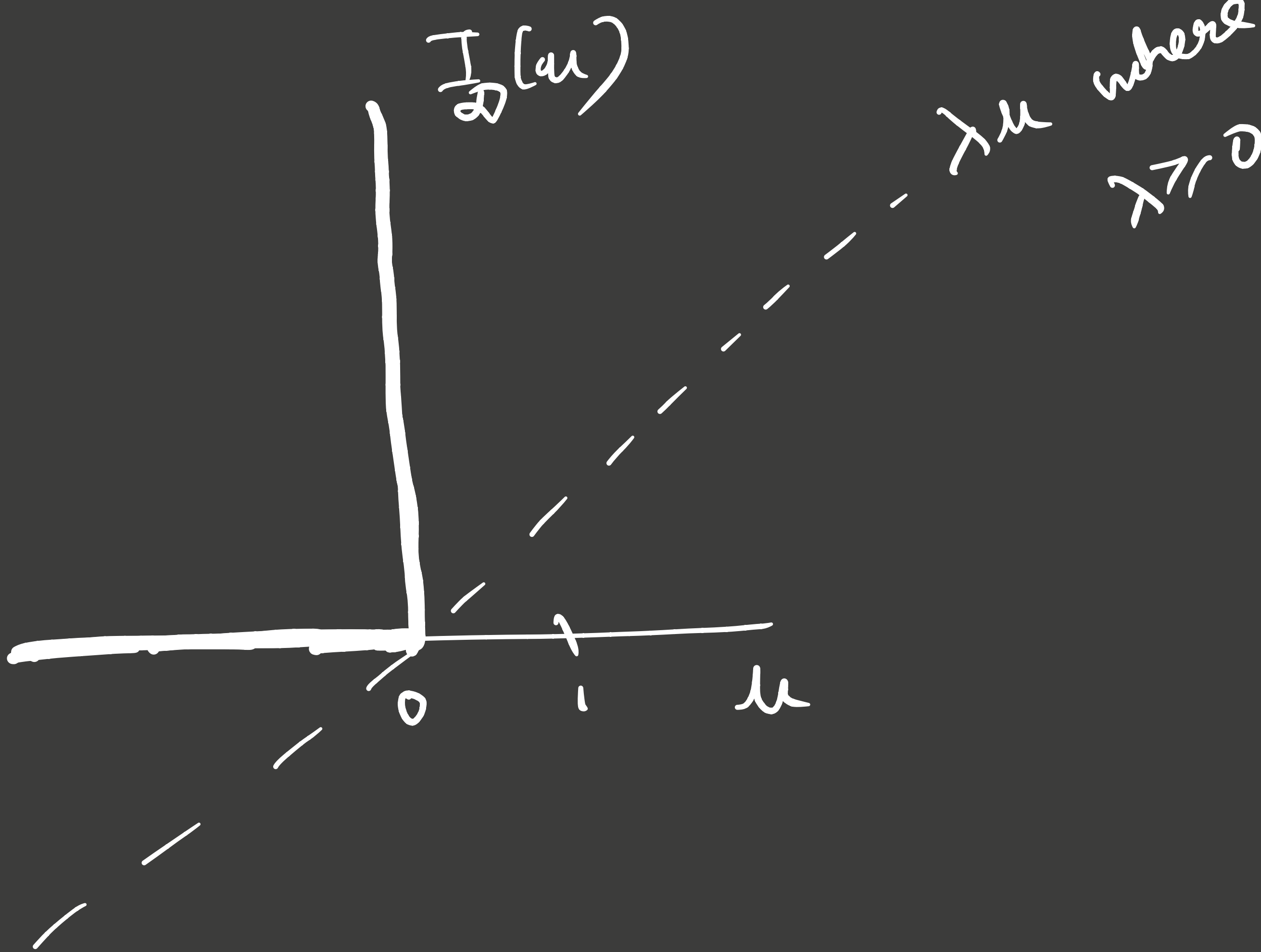
Approximate $I_\infty(u)$ with λu $(\lambda \geq 0)$



$\lambda \geq 0$

$\therefore \lambda u \geq 0$ for $u \geq 0$ $\rightarrow$ Add penalty

Also λu is lower bound on $I_\infty(u)$



lower bound
Every element $\leq$ other set

Appx. function of $J(x)$

$$L(x,\lambda) = x^2 + \lambda(x-2)$$

Q) Relationship b/w $L(x,\lambda)$ and $J(x)$

Case I

$$x - 2 \leq 0$$

$$\lambda = 0$$

Case II

$$x - 2 > 0$$

$$\lambda \to \infty$$

$$\therefore \max_{\lambda} L(x,\lambda) = J(x)$$

$$\max_{\lambda} L(x,\lambda) = J(x)$$

Original problem:

Minimize $J(x)$

or

$$\min_{x} \max_{\lambda} L(x,\lambda)$$

or

$$\max_{\lambda} \min_{x} L(x,\lambda) \quad \text{or} \quad \max_{\lambda} g(\lambda)$$

(under certain conditions)

$g(\lambda)$: Dual function

$L(x,\lambda)$ = lower bound on $J(x)$

$\Rightarrow \quad L(x,\lambda) \leq J(x)$

$$\min_x L(x,\lambda) = g(\lambda) \leq \min_x J(x) = p^*$$

$$d^* = \max_\lambda g(\lambda) \leq p^*$$

p^* : Primal optima

d^* : Dual optima

For convex problems,

$$d^* = p^*$$

$\therefore$ Minimize Primal $\Longleftrightarrow$ Maximize dual