Convention, Accuracy metrics, Classification, Regression

Nipun Batra January 9, 2020

IIT Gandhinagar

Please sign up on Google Classroom: cgt6rwd

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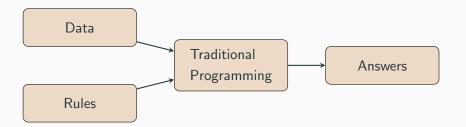
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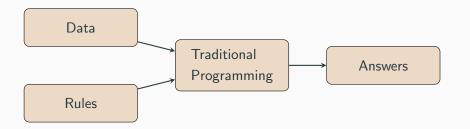
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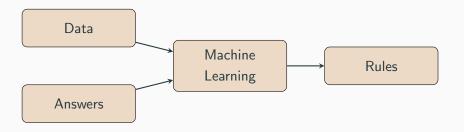
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"A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P if its performance at tasks in T, as measured by P, improves with experience E." - Tom Mitchell

First ML Task: Grocery store tomatoes quality prediction

Problem statement: You want to predict the quality/condition of a tomato given its visual features.

Imagine you have some past data on quality of tomatoes. What visual features do you think will be useful?

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Size

Imagine you have some past data on quality of tomatoes. What visual features do you think will be useful?

- Size
- Colour

Imagine you have some past data on quality of tomatoes. What visual features do you think will be useful?

- Size
- Colour
- Texture

Imagine you have some past data on quality of tomatoes.

Sample	Colour	Size	Texture	Condition
1	Orange	Small	Smooth	Good
2	Red	Small	Rough	Good
3	Orange	Medium	Smooth	Bad
4	Yellow	Large	Smooth	Bad

Useful Features

Is the sample number a useful feature for predicting quality of a tomato?

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Answer: It depends! Maybe, all tomatoes received after a certain date are bad! Let us ignore that for now.

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Let us modify our data table for now.

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The training set consists of two parts:

Colour	Size	Texture	Condition
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1. Features, Attributes or Covariates

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The training set consists of two parts:

- 1. Features, Attributes or Covariates
- 2. Output or Response Variable

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We call this matrix as $\ensuremath{\mathcal{D}}\xspace$, containing:

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We call this matrix as \mathcal{D} , containing:

1. Feature matrix $(\mathbf{X} \in \mathcal{R}^{\mathbf{N} \times \mathbf{P}})$ containing data of N samples each of which is P dimensional.

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• Example $x_1 = \begin{bmatrix} Orange \\ Small \\ Smooth \end{bmatrix}$

Training Set

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2. Output Vector $(y \in \mathcal{R}^N)$ containing output variable for N samples.

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- 2. Output Vector $(y \in \mathcal{R}^N)$ containing output variable for N samples.
- 3. Thus, we can also write $\mathcal{D} = \{(x_i^T, y_i)\}_{i=1}^N$

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Estimate condition for unseen tomatoes (#5, 6) based on data set.

Colour	Size	Texture	Condition
Orange	Small	Smooth	Good
Red	Small	Rough	Good
Orange	Medium	Smooth	Bad
Yellow	Large	Smooth	Bad
Red	Large	Rough	?
Orange	Large	Rough	?

Testing Set

Testing set is similar to training set, but, does not contain labels for output variable.

Colour	Size	Texture	Condition
Orange	Small	Smooth	Good
Red	Small	Rough	Good
Orange	Medium	Smooth	Bad
Yellow	Large	Smooth	Bad
Red	Large	Rough	?
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- 3. To Predict the condition for the Testing set

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- Q: Is predicting on test set enough to say our model generalises?
- A: Ideally, no!
- Ideally we want to predict "well" on all possible inputs. But, can we test that?
- No! Since, the test set is only a sample from all possible inputs.

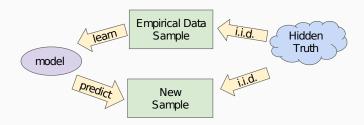


Image courtesy Google ML crash course

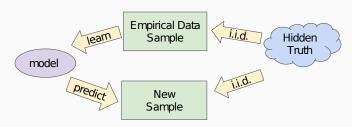


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Both the training set and the test set are samples drawn from the hidden true distribution (also sometimes called population)

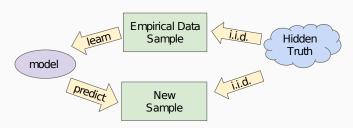


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More discussion later once we study bias and variance

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- • # People (More people ⇒ More Energy)
- Temperature (Higher Temp. \implies Higher Energy)

# People	Temp (C)	Energy (kWh)
4000	30	30
4200	30	32
4200	35	40
3000	20	?
1000	45	?

Classification

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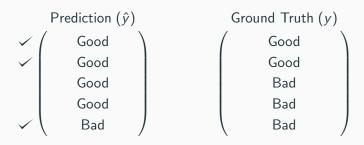
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 - Output variable is continuous
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 - Examples Predicting:
 - How much energy will campus consume?
 - How much rainfall will fall?

Metrics for Classification

Ground Truth: From the actual training set

Prediction: Made by the model

Accuracy



Accuracy

Prediction
$$(\hat{y})$$
 Ground Truth (y)
 \checkmark Good
Good
Good
Good
Bad
Bad
Bad
Bad

$$\begin{aligned} \mathsf{Accuracy} &= \frac{||y = \hat{y}||}{||y||} \\ &= \frac{3}{5} = 0.6 \end{aligned}$$

Types of Data: Imbalanced Classes

$$\begin{array}{c} 1 \; \mathsf{sample} \; \{ \; \left(\begin{array}{c} \mathsf{Bad} \\ \mathsf{Good} \\ \mathsf{Good} \\ \dots \\ \mathsf{Good} \end{array} \right) \\ & \mathsf{Imbalanced} \; \mathsf{Classes} \end{array}$$

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Cases for this:

- Cancer Screening
- Planet Detection

Accuracy Metrics: Precision

Precision =
$$\frac{||y = \hat{y} = Good||}{||\hat{y} = Good||} = \frac{2}{4} = 0.5$$

"the fraction of relevant instances among the retrieved instances", i.e. "out of the number of times we predict Good, how many times is the condition actually Good"

Accuracy Metrics: Precision

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Accuracy Metrics: Recall

Recall =
$$\frac{||y = \hat{y} = \text{Good}||}{||y = \text{Good}||} = \frac{2}{3} = 0.67$$

"the fraction of the total amount of relevant instances that were actually retrieved"

Types of Data: Imbalanced Classes

Given predictions of whether a tissue is cancerous or not (n = 100).

Types of Data: Imbalanced Classes

Given predictions of whether a tissue is cancerous or not (n = 100).

$$\mbox{Accuracy} = \frac{98}{100} = 0.98 \qquad \qquad \mbox{Recall} = \frac{0}{1} = 0$$

$$\mbox{Precision} = \frac{0}{1} = 0$$

		Ground Truth	
		Yes	No
cted	Yes	0	1
redicted	No	1	98
Д			

		Ground Truth	
		Yes	No
ted	Yes	0	1
redicted	No	1	98
_			

		Ground Truth	
		Yes	No
ted	Yes	True Positive	False Positive
redicted	No	False Negative	True Negative
Д.			

		Ground Truth	
		Yes	No
redicted	Yes	True Positive	False Positive
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$$Precision = \frac{T.P.}{T.P.+F.P.}$$

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Д			

$$Recall = \frac{T.P.}{T.P.+F.N.}$$

		Ground Truth		
		Yes	No	
cted	Yes	True Positive	False Positive	
redicted	No	False Negative	True Negative	
Д				

Recall =
$$\frac{T.P.}{T.P.+F.N.}$$

Accuracy Metrics: F-Score

		Ground Truth	
		Yes	No
cted	Yes	True Positive	False Positive
redicted	No	False Negative	True Negative
\Box			

$$F-$$
 Score = $\frac{2 \times Precision \times Recall}{Precision + Recall}$

Accuracy Metrics: Matthew's Correlation Coefficient

		Ground Truth	
		Yes	No
cted	Yes	True Positive	False Positive
redicted	No	False Negative	True Negative
Д			

$$\frac{\text{TP} \times \text{TN} - \text{FP} \times \text{FN}}{\sqrt{(\text{TP} + \text{FP})(\text{TP} + \text{FN})(\text{TN} + \text{FP})(\text{TN} + \text{FN})}}$$

Accuracy Metrics: Example

For the data given below, calculate:

$$\begin{array}{cccc} & & G.T. \ \mbox{Positive} & G.T. \ \mbox{Negative} \\ \mbox{Pred Positive} & & 90 & 4 \\ \mbox{Pred Negative} & & 1 & 1 \\ \end{array}$$

Precision = ?

Recall = ?

F-Score = ?

Matthew's Coeff. =?

Accuracy Metrics: Answer

For the same data

G.T. Positive G.T. Negative Pred Positive
$$\begin{pmatrix} 90 & 4 \\ 1 & 1 \end{pmatrix}$$

Precision =
$$\frac{90}{94}$$

Recall = $\frac{90}{91}$
F-Score = 0.9524
Matthew's Coeff. = 0.14

Metrics for Regression MSE & MAE

Prediction
$$(\hat{y})$$
 Ground Truth (y)

$$\begin{pmatrix}
10 \\
20 \\
30 \\
40 \\
50 \\
60
\end{pmatrix}$$

Mean Squared Error (MSE) =
$$\frac{\sum_{i=1}^{N} (\hat{y}_i - y_i)^2}{N}$$
Root Mean Square Error (RMSE) = $\sqrt{\text{MSE}}$

Accuracy Metrics: MAE & ME

Prediction
$$(\hat{y})$$
 Ground Truth

 $\begin{pmatrix}
10 & & & \\
20 & & & \\
30 & & & \\
40 & & & \\
50 & & & & \\
60 & & & & \\
\end{pmatrix}$

Mean Absolute Error (ME) =
$$\frac{\sum_{i=1}^{N} |\hat{y}_i - y_i|}{N}$$
Mean Error =
$$\frac{\sum_{i=1}^{N} \hat{y}_i - y_i}{N}$$

Accuracy Metrics: MAE & ME

Prediction
$$(\hat{y})$$
 Ground Truth

 $\begin{pmatrix}
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Is there any downside with using mean error?

Accuracy Metrics: MAE & ME

Prediction
$$(\hat{y})$$
 Ground Truth

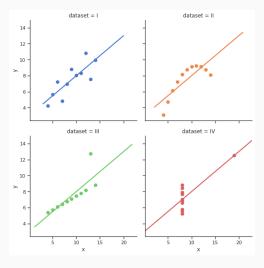
 $\begin{pmatrix}
10 \\
20 \\
30 \\
40 \\
50
\end{pmatrix}$
Ground Truth

 $\begin{pmatrix}
60 \\
60 \\
60
\end{pmatrix}$

Mean Absolute Error (ME) =
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Is there any downside with using mean error? Errors can get cancelled out

The Importance of Plotting



Anscombe's Quartet

The Importance of Plotting

Property	Value	Accross datasets
mean(X)	9	exact
mean(Y)	7.5	upto 3 decimal places
Linear regression line	y = 3.00 + 0.500x	upto 2 decimal places

Try to play with the colab link to see how similar the metrics like variance and correlation are.