# Convention, Accuracy metrics, Classification, Regression

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"A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P if its performance at tasks in T, as measured by P, improves with experience E." - Tom Mitchell

Problem statement: You want to predict the quality/condition of a tomato given its visual features.



- Size
- Colour

- Size
- Colour
- Texture

Imagine you have some past data on quality of tomatoes.

Sample	Colour	Size	Texture	Condition
1	Orange	Small	Smooth	Good
2	Red	Small	Rough	Good
3	Orange	Medium	Smooth	Bad
4	Yellow	Large	Smooth	Bad

Is the sample number a useful feature for predicting quality of a tomato?

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Answer: It depends! Maybe, all tomatoes received after a certain date are bad! Let us ignore that for now.

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Let us modify our data table for now.

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Red	Small	Rough	Good
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The training set consists of two parts:

Colour	Size	Texture	Condition
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Orange	Medium	Smooth	Bad
Yellow	Large	Smooth	Bad

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1. Features, Attributes or Covariates

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The training set consists of two parts:

- 1. Features, Attributes or Covariates
- 2. Output or Response Variable

Colour	Size	Texture	Condition
Orange	Small	Smooth	Good
Red	Small	Rough	Good
Orange	Medium	Smooth	Bad
Yellow	Large	Smooth	Bad

Colour	Size	Texture	Condition
Orange	Small	Smooth	Good
Red	Small	Rough	Good
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We call this matrix as  $\mathcal{D}$ , containing:

Colour	Size	Texture	Condition
Orange	Small	Smooth	Good
Red	Small	Rough	Good
Orange	Medium	Smooth	Bad
Yellow	Large	Smooth	Bad

We call this matrix as  $\mathcal{D}$ , containing:

1. Feature matrix  $(X \in \mathcal{R}^{N \times P})$  containing data of N samples each of which is P dimensional.

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• Thus, 
$$X = \{x_i^T\}_{i=1}^N$$
 where  $x_i \in \mathcal{R}^P$ 

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• Example  $x_1 = \begin{bmatrix} Orange \\ Small \\ Smooth \end{bmatrix}$ 

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$$X = \{x_i^T\}_{i=1}^N$$
 where  $x_i \in \mathcal{R}^N$   
• Example  $x_1 = \begin{bmatrix} Orange \\ Small \\ Smooth \end{bmatrix}$ 

2. Output Vector  $(y \in \mathcal{R}^N)$  containing output variable for N samples.

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Orange	Small	Smooth	Good
Red	Small	Rough	Good
Orange	Medium	Smooth	Bad
Yellow	Large	Smooth	Bad

We call this matrix as  $\mathcal{D}$ , containing:

- Feature matrix (X ∈ R<sup>N×P</sup>) containing data of N samples each of which is P dimensional.
  - Thus,  $X = \{x_i^T\}_{i=1}^N$  where  $x_i \in \mathcal{R}^P$ • Example  $x_1 = \begin{bmatrix} Orange \\ Small \\ Smooth \end{bmatrix}$
- Output Vector (y ∈ R<sup>N</sup>) containing output variable for N samples.
- 3. Thus, we can also write  $\mathcal{D} = \{(x_i^T, y_i)\}_{i=1}^N$

Estimate condition for unseen tomatoes (#5, 6) based on data set.

Colour	Size	Texture	Condition
Orange	Small	Smooth	Good
Red	Small	Rough	Good
Orange	Medium	Smooth	Bad
Yellow	Large	Smooth	Bad
Red	Large	Rough	?
Orange	Large	Rough	?

Testing set is similar to training set, but, does not contain labels for output variable.

Colour	Size	Texture	Condition
Orange	Small	Smooth	Good
Red	Small	Rough	Good
Orange	Medium	Smooth	Bad
Yellow	Large	Smooth	Bad
Red	Large	Rough	?
Orange	Large	Rough	?

## **Prediction Task**

We hope to:
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1. Learn f: Condition = f(colour, size, texture)

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- 2. From Training Dataset

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- 1. Learn f: Condition = f(colour, size, texture)
- 2. From Training Dataset
- 3. To Predict the condition for the Testing set

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Orange	Small	Smooth	Good
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Yellow	Large	Smooth	Bad
Red	Large	Rough	?
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- Q: Is predicting on test set enough to say our model generalises?
- A: Ideally, no!
- Ideally we want to predict "well" on all possible inputs. But, can we test that?
- No! Since, the test set is only a sample from all possible inputs.



Image courtesy Google ML crash course



Image courtesy Google ML crash course

Both the training set and the test set are samples drawn from the hidden true distribution (also sometimes called population)



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More discussion later once we study bias and variance

#### Second ML Task: Predict energy consumption of campus

Question: What factors does the campus energy consumption depend on?

Answer:

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Answer:

- # People (More people  $\implies$  More Energy)
- Temperature (Higher Temp.  $\implies$  Higher Energy)

# People	Temp (C)	Energy (kWh)
4000	30	30
4200	30	32
4200	35	40
3000	20	?
1000	45	?

• Classification

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  - Output variable is discrete

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  - Examples Predicting:
    - Will I get a loan? (Yes, No)

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    - Will I get a loan? (Yes, No)
    - What is the quality of fruit? (Good, Bad)

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  - i.e.  $y_i \in \mathcal{R}$

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  - Examples Predicting:
    - How much energy will campus consume?

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    - What is the quality of fruit? (Good, Bad)
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  - Output variable is continuous
  - i.e.  $y_i \in \mathcal{R}$
  - Examples Predicting:
    - How much energy will campus consume?
    - How much rainfall will fall?



Ground Truth: From the actual training set Prediction: Made by the model Accuracy



Accuracy



$$\begin{aligned} \mathsf{Accuracy} &= \frac{||y = \hat{y}||}{||y||} \\ &= \frac{3}{5} = 0.6 \end{aligned}$$

#### Types of Data: Imbalanced Classes



Imbalanced Classes

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Imbalanced Classes

Cases for this:

- Cancer Screening
- Planet Detection



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$$\text{Recall} = \frac{||y = \hat{y} = \text{Good}||}{||y = \text{Good}||} = \frac{2}{3} = 0.67$$

"the fraction of the total amount of relevant instances that were actually retrieved"

# **Types of Data: Imbalanced Classes**

Given predictions of whether a tissue is cancerous or not (n = 100).



#### **Types of Data: Imbalanced Classes**

Given predictions of whether a tissue is cancerous or not (n = 100).



Accuracy 
$$=$$
  $\frac{98}{100} = 0.98$  Recall  $=$   $\frac{0}{1} = 0$   
Precision  $=$   $\frac{0}{1} = 0$
# **Accuracy Metrics: Confusion Matrix**

		Ground Truth	
		Yes	No
cted	Yes	0	1
redic	No	1	98
Д_			

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$$\mathsf{Precision} = \frac{T.P.}{T.P.+F.P.}$$



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$$\mathsf{Recall} = \frac{T.P.}{T.P.+F.N.}$$



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$$F-Score = \frac{2 \times Precision \times Recall}{Precision + Recall}$$

		Ground Truth	
		Yes	No
cted	Yes	True Positive	False Positive
redic	No	False Negative	True Negative
Δ_			

 $\begin{array}{l} \text{Matthew's correlation coefficient} = \\ \frac{\text{TP} \times \text{TN} - \text{FP} \times \text{FN}}{\sqrt{(\text{TP} + \text{FP})(\text{TP} + \text{FN})(\text{TN} + \text{FP})(\text{TN} + \text{FN})}} \end{array}$ 

For the data given below, calculate:

$$\begin{array}{c} \text{G.T. Positive} \quad \text{G.T. Negative} \\ \text{Pred Positive} \begin{pmatrix} 90 & 4 \\ 1 & 1 \end{pmatrix} \end{array}$$

Precision = ? Recall = ? F-Score = ? Matthew's Coeff. = ?

#### For the same data

G.T. Positive G.T. Negative  
Pred Positive 
$$\begin{pmatrix} 90 & 4 \\ 1 & 1 \end{pmatrix}$$

 $\begin{array}{l} \mbox{Precision} = \frac{90}{94} \\ \mbox{Recall} = \frac{90}{91} \\ \mbox{F-Score} = 0.9524 \\ \mbox{Matthew's Coeff.} = 0.14 \end{array}$ 



Mean Squared Error (MSE) =  $\frac{\sum_{i=1}^{N} (\hat{y}_i - y_i)^2}{N}$ Root Mean Square Error (RMSE) =  $\sqrt{MSE}$ 

### Accuracy Metrics: MAE & ME

Prediction $(\hat{y})$	Ground Truth
( 10 )	$\begin{pmatrix} 20 \end{pmatrix}$
20	30
30	40
40	50
50	60

Mean Absolute Error (ME) =  $\frac{\sum_{i=1}^{N} |\hat{y}_i - y_i|}{N}$ Mean Error =  $\frac{\sum_{i=1}^{N} \hat{y}_i - y_i}{N}$ 

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Is there any downside with using mean error?

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Is there any downside with using mean error? Errors can get cancelled out

# The Importance of Plotting



Anscombe's Quartet

Property	Value	Accross datasets
mean(X)	9	exact
mean(Y)	7.5	upto 3 decimal places
Linear regression line	y = 3.00 + 0.500x	upto 2 decimal places

Try to play with the colab link to see how similar the metrics like variance and correlation are.