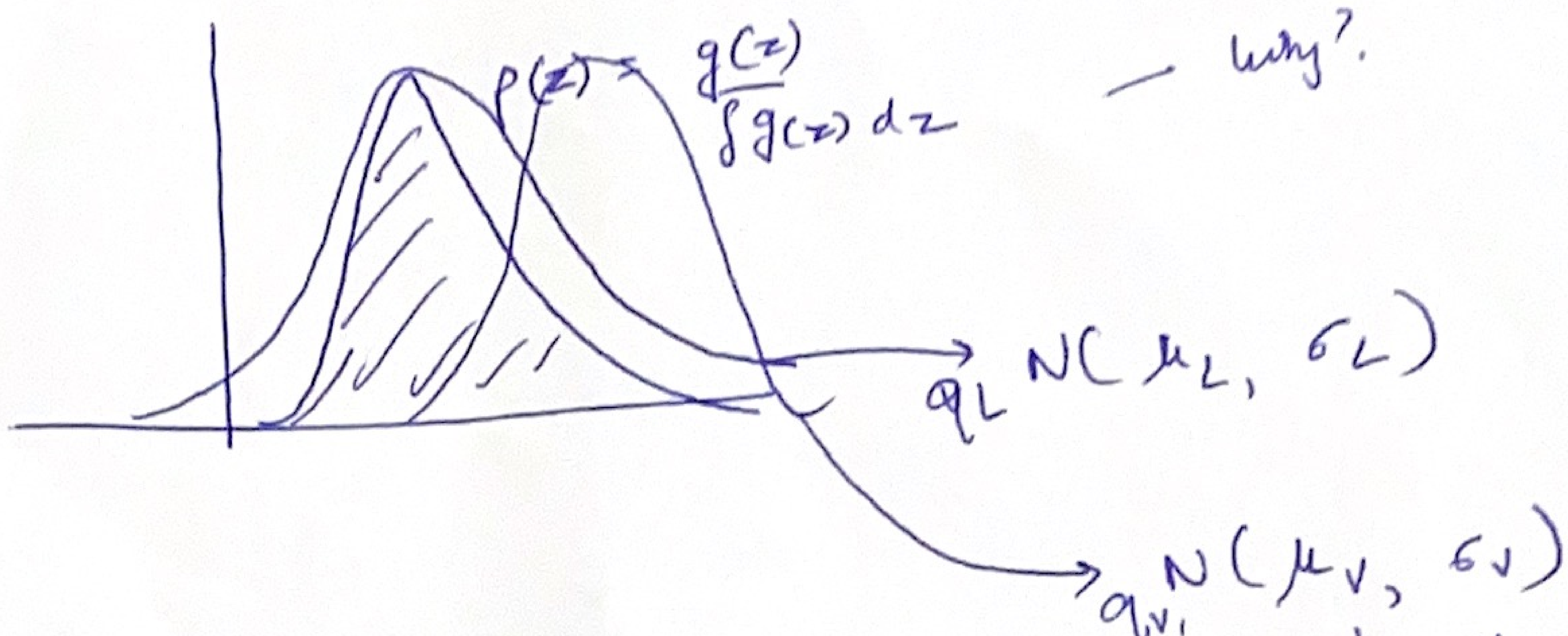
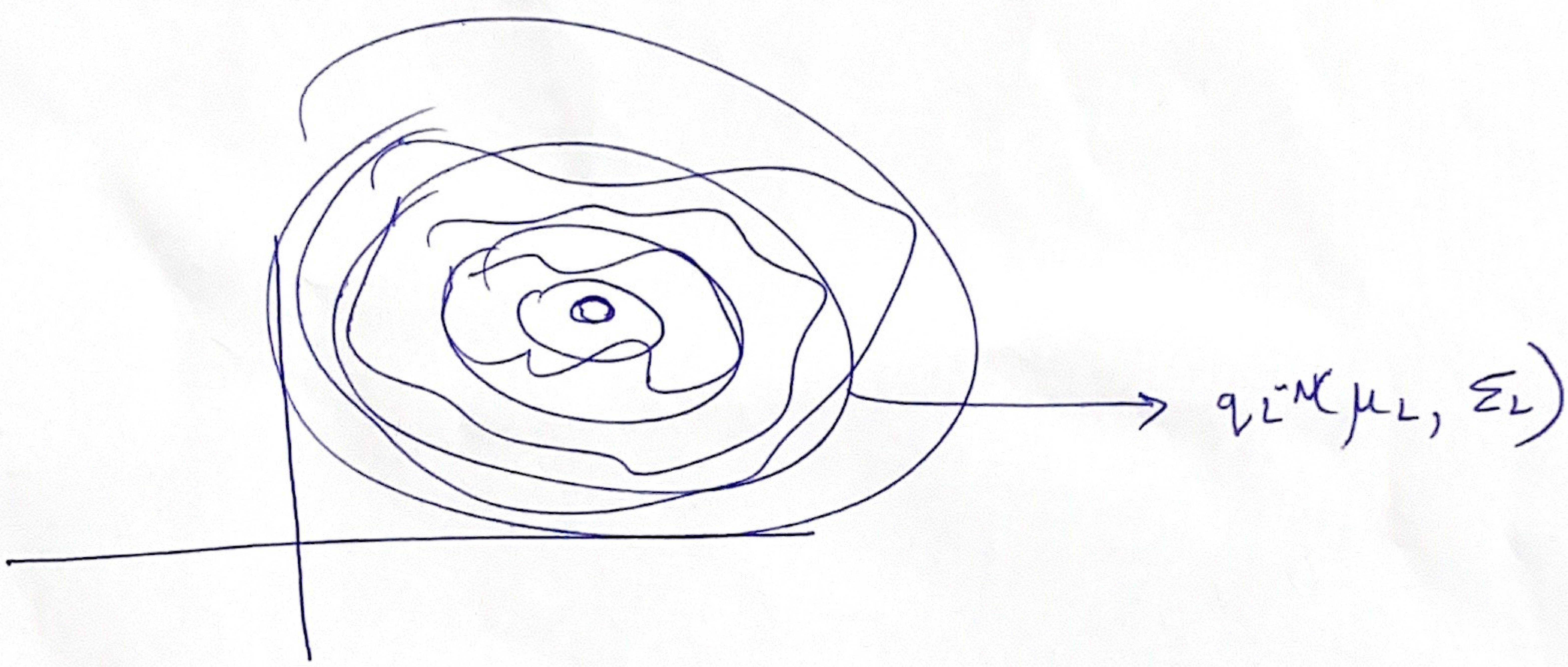
Laplace Approximation:



Variational inference.

Minimize "divergence" b/w q_v and $p(z)$



Why? (Bayes. Logistic Regression)

$$p(\theta|D) = ? = \frac{p(D|\theta) \cdot p(\theta)}{p(D)}$$

$$p(D) = \int p(D|\theta)\, p(\theta)\, d\theta = \int \prod_{i=1}^{N} \sigma(x_i^T\theta)^{y_i} \left(1 - \sigma(x_i^T\theta)\right)^{1-y_i} N(0, b^2 I)\, d\theta$$

Taylor Series

Appn $f(x) = \cos x$ around $x = 0$ w/ Polynomial
$= x_0$

$g(x) = c_0 + c_1 x + c_2 x^2 + \dots$

$$g(x) = f(0) + \frac{f'(x)}{1!}\Big|_{x=0} x + \frac{f''(x)}{2!}\Big|_{x=0} x^2 + \dots$$

General form

Appn $f(x)$ at $x = x_0$ w/ Polynomial

$$g(x) = f(x_0) + \frac{f'(x)\big|_{x=x_0}}{1}(x - x_0) + \frac{f''(x)\big|_{x=x_0}}{2!}(x - x_0)^2 + \dots$$

Ques $f(x) = \cos x$ around $x = \pi$

Multivariate form

$$g(x) = f(x_0) + (x - x_0)^T \nabla f(x_0) \cancel{(x-x_0)} + \frac{1}{2!}(x - x_0)^T \nabla^2 f(x_0)(x - x_0) + \dots$$

$$f(x) = \cos x_1 + \cos x_2$$

$$\nabla f = \begin{bmatrix} -\sin x_1 \\ -\sin x_2 \end{bmatrix}_{0,0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\nabla^2 f = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 x_2} \\ \frac{\partial^2 f}{\partial x_2 x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x_1}(-\sin x_1) & \frac{\partial}{\partial x_1}(-\sin x_2) \\ \frac{\partial}{\partial x_2}(-\sin x_1) & \frac{\partial}{\partial x_2}(-\sin x_2) \end{bmatrix}$$

$$= \begin{bmatrix} -\cos x_1 & 0 \\ 0 & -\cos x_2 \end{bmatrix}_{0,0}$$

$$\therefore\ g(x) = \cos 0 \overset{\cos 0}{+} x^T \begin{bmatrix} 0 \\ 0 \end{bmatrix} x + \frac{1}{2} x^T \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} x$$

$$= 2 - \frac{1}{2} . x^T x$$

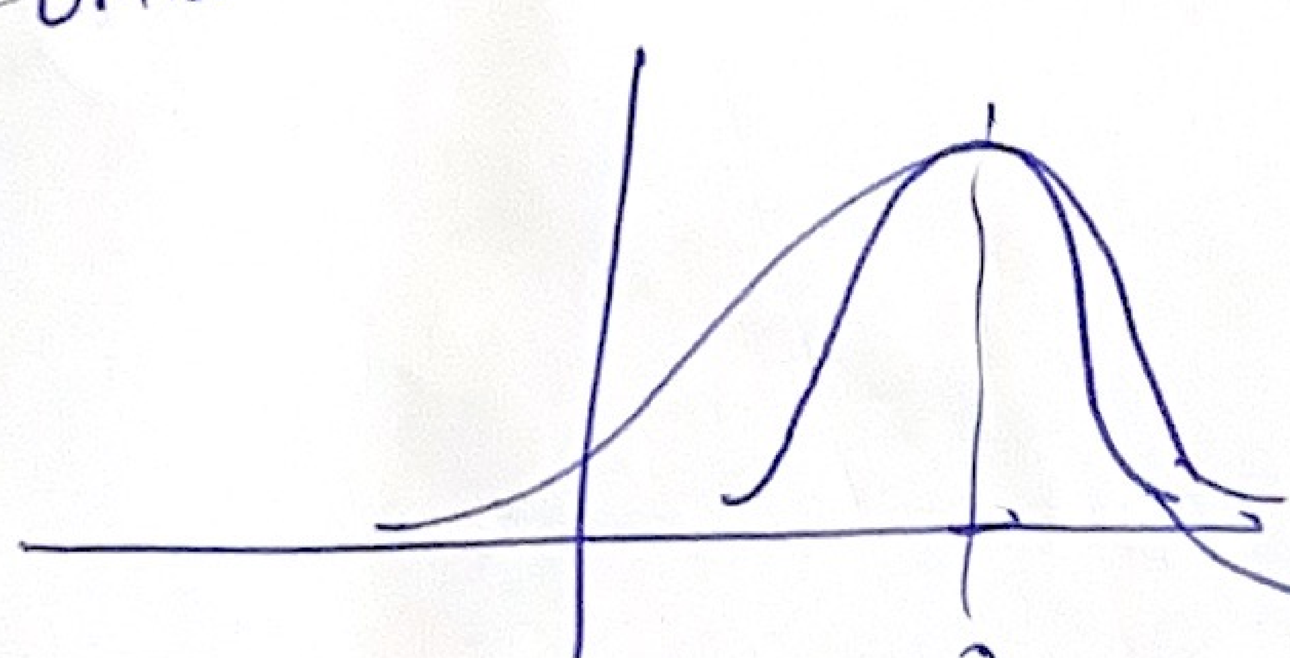
$$P(\theta|D) = ? = \frac{P(D,\theta)}{\int P(D,\theta)\,d\theta}$$

$$= \frac{e^{\log P(D,\theta)}}{\int e^{\log P(D,\theta)}\,d\theta}$$

$$\log P(D,\theta) = f(\theta)$$

$$f(\theta) \approx f(\theta_0) + (\theta-\theta_0)^T \nabla f(\theta_0)\,(\theta-\theta_0)^2 + \dots$$

$\theta_0 = \theta_{MAP}$



θ_{MAP} $\rightarrow$ Appro $\approx N(\theta_{MAP}, ?)$

$$\nabla f(\theta_{MAP}) = 0$$

$$f(\theta) \approx f(\theta_{MAP}) + \frac{1}{2}(\theta-\theta_{MAP})^T \nabla^2 f(\theta_{MAP})(\theta-\theta_{MAP})$$

$$= \log P(D, \theta_{MAP}) + \dots$$

$$\therefore\ P(\theta|D) = \frac{e^{\log(D,\theta_{MAP})} \cdot e^{-\frac{1}{2}(\theta-\theta_{MAP})^T(-\nabla^2 f(\theta_{MAP}))(\dots)}}{\int \dots \cdot \dots}$$

$$= N(\theta \mid \theta_{MAP}, H^{-1})$$

$$H^{-1} = -\nabla^2 \log(D, \theta_{MAP})$$

For Log. Reg.:

$$P(\theta) = N(0, \sigma^2 I)$$

$$P(D \mid \theta) =$$