$$P(\theta|D) = \frac{P(D|\theta)P(\theta)}{\int P(D|\theta)P(\theta)d\theta}$$

(i) $\hat{\theta}_{MLE}$: POINT

(ii) $\hat{\theta}_{MAP}$ = POINT

(iii) $P(\theta|D)$: DI   IBUTION:

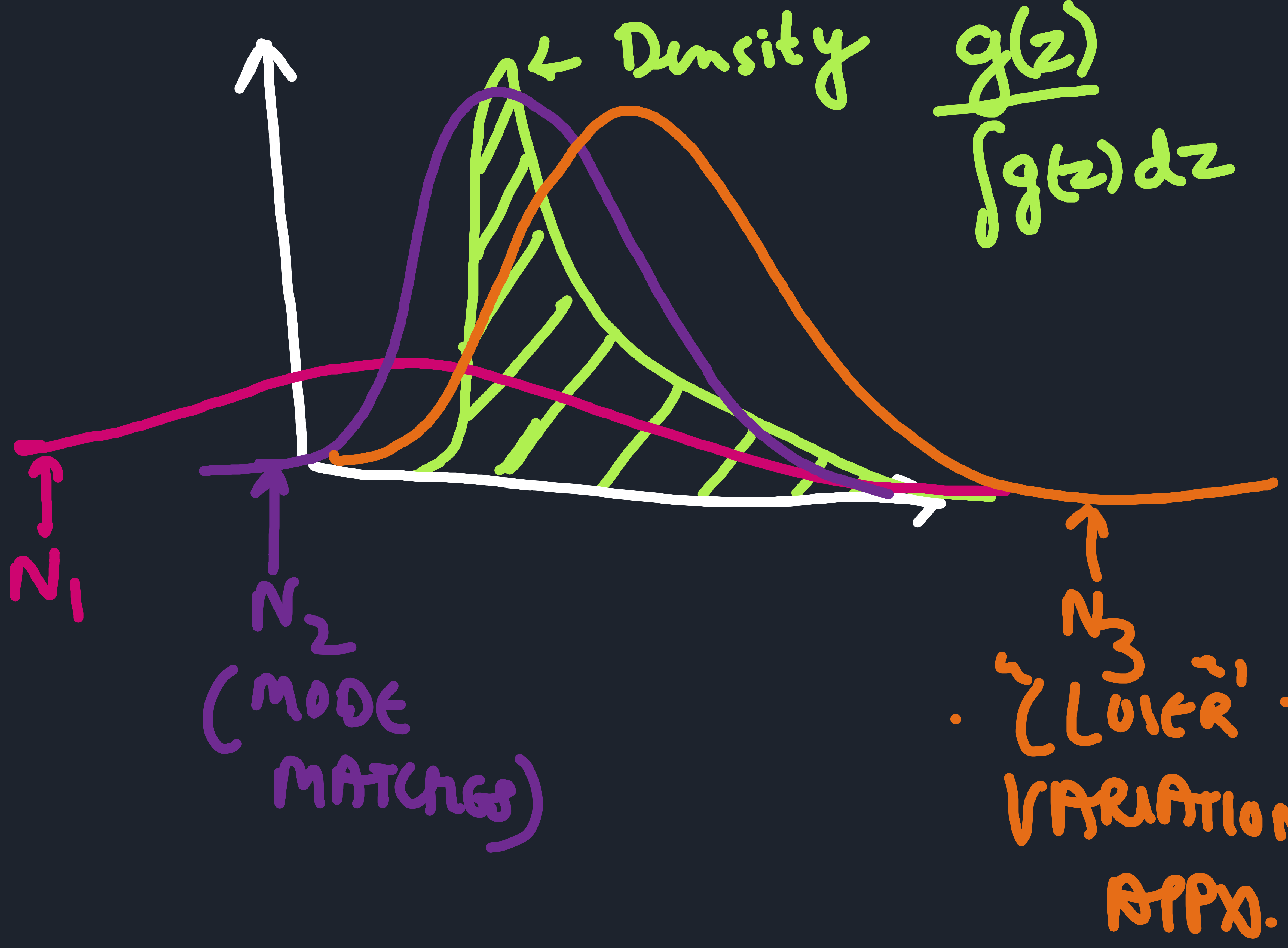
DENOMINATOR

FOR Lin Reg.

$$\int P(D|\theta)P(\theta)\,d\theta$$

NORMAL   NORMAL

Laplace Appx.



Approximate density w/ NORMAL

$$\mathcal{N}\left(\begin{bmatrix}\mu_0\\ \mu_1\end{bmatrix}, \Sigma\right) \quad ; \begin{bmatrix}\mu_0\\ \mu_1\end{bmatrix} = \text{MODE}(\text{TARGET})$$

Taylor Series

Appx $f(x)$. eg $\cos(x)$ around $x=0$

$$g(x) = c_0 + c_1 x + c_2 x^2 + \dots$$

$$g(x) = f(0) + \frac{f'(x)\big|_{x=0}}{1!} * x + \frac{f''(x)\big|_{x=0}}{2!} * x^2 + \dots$$

Appx. $f(x)$ arond $x = x_0$

$$g(x) = f(x_0) + \frac{f'(x)\Big|_{x=x_0}}{1!} \times (x - x_0) + \frac{f''(x)\Big|_{x=x_0}}{2!} (x-x_0)^2 + \dots$$

Q) $f(x) = \cos(x)$ arond $x = \pi$

$$= \cos(\pi) - \frac{\sin(\pi)}{1!}(x-\pi) - \frac{\cos(\pi)}{2!}(x-\pi)^2 + \dots$$

$$= -1 + \frac{1}{2}(x-\pi)^2 + \dots$$

Taylor Series for M.V. Input at $x=x_0$

$$g(x) = f(x_0) + \frac{1}{1!}(x-x_0)^T \nabla f(x_0) + \frac{1}{2!}(x-x_0)^T \nabla^2 f(x_0)(x-x_0) + \dots$$

Q) $f(x) = \cos x_1 + \cos x_2$

around $x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\cos 0 + \cos 0 + \frac{1}{1!}\begin{bmatrix} x_1 & x_2 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} + \frac{1}{2!}\begin{bmatrix} x_1 & x_2 \end{bmatrix} \nabla^2 f\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\nabla^2 f\left(\begin{bmatrix}0\\0\end{bmatrix}\right) = \begin{bmatrix} \dfrac{\partial^2 f}{\partial x_1^2} & \dfrac{\partial^2 f}{\partial x_1 \partial x_2} \\[2ex] \dfrac{\partial^2 f}{\partial x_2 \partial x_1} & \dfrac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\partial}{\partial x_1}(-\sin x_1) & \dfrac{\partial}{\partial x_1}(-\sin x_2) \\[2ex] \dfrac{\partial}{\partial x_2}(-\sin x_1) & \dfrac{\partial}{\partial x_2}(-\sin x_2) \end{bmatrix} = \begin{bmatrix} -\cos x_1 & 0 \\ 0 & -\cos x_1 \end{bmatrix}\Bigg|_{0,\,0} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\therefore\ g(x) = 1+1-\frac{1}{2}\begin{bmatrix} x_1 & x_2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= 2-\frac{1}{2}\begin{bmatrix} x_1 & x_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= 2-\frac{1}{2}(x_1^2+x_2^2)$$

$$\theta_{MAP} = \arg\max_{\theta} P(\theta | D)$$

$$P(\theta | D) = ?$$

$$= \frac{P(D|\theta)P(\theta)}{P(D)} = \frac{P(D,\theta)}{\int P(D,\theta)\,d\theta}$$

$$= \frac{e^{\log P(D,\theta)}}{\int e^{\log P(D,\theta)}\,d\theta} \longrightarrow f(\theta) = \log P(D,\theta)$$

$$\Theta_{MAP} = \arg\max_{\theta} P(\theta \mid D) = \arg\max_{\theta} \underbrace{P(D \mid \theta) P(\theta)}_{P(D,\theta)}$$

$$f(\theta) = \log P(D, \theta)$$

Appx. w/ polynomial around $\theta = \Theta_0$

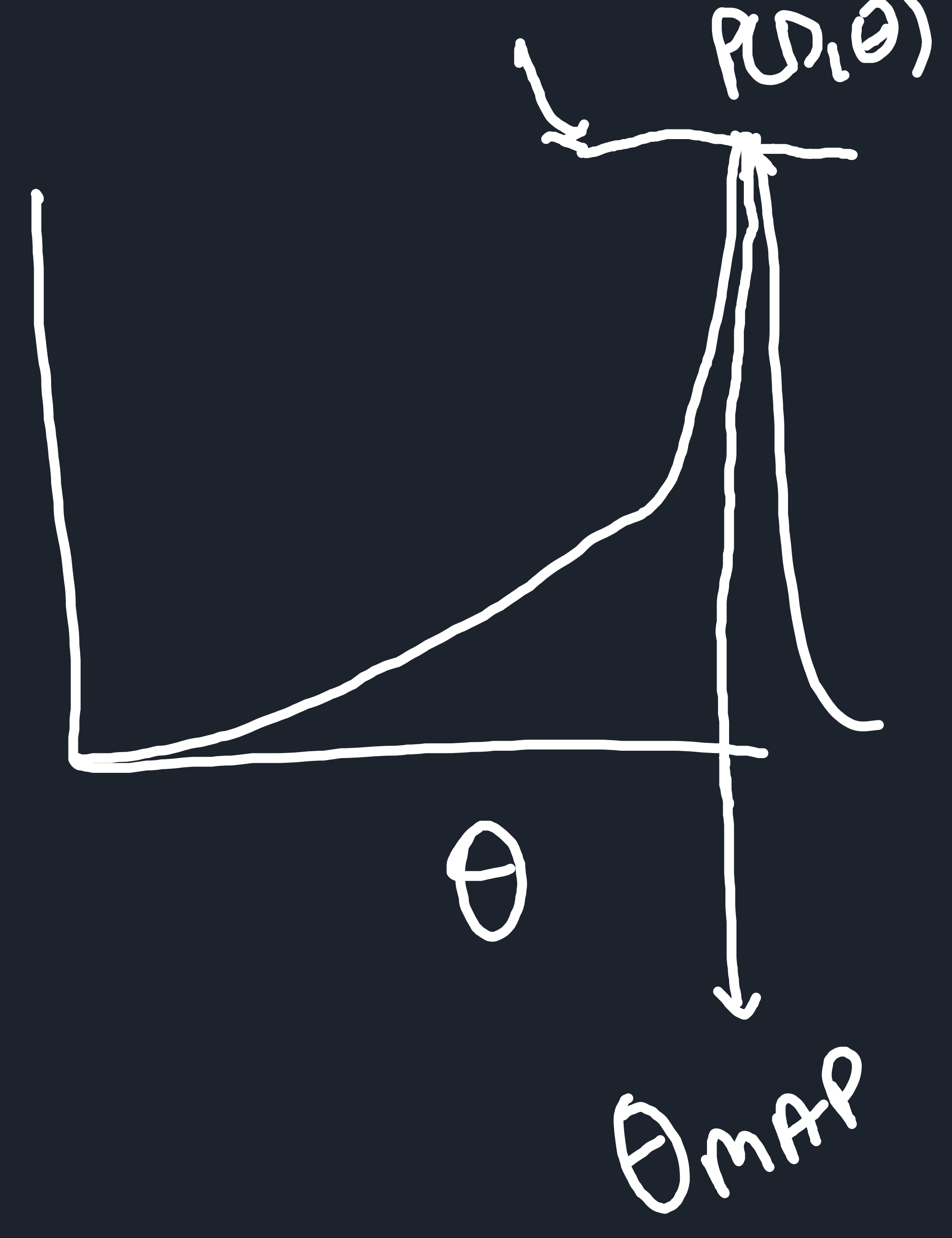
$$f(\theta) = \log(D, \Theta_0) + 1(\theta - \theta_0)^T \nabla f\big|_{\Theta_0} + \frac{1}{2}(\theta - \Theta_0)^T \nabla^2 f\big|_{\theta_0}(\theta - \Theta_0) + \dots$$

At $\theta = \theta_{MAP}$

$$\nabla f\big|_{\theta_{MAP}} = 0$$

$\therefore f(\theta)$ around $\theta = \Theta_{MAP}$

$$= \log(D, \Theta_0) - \frac{1}{2}(\theta - \theta_0)^T(-\nabla^2 f\big|_{\theta_{MAP}})(\theta - \theta_0)$$



$$\nabla f\big|_{\theta = \theta_{MAP}} = 0$$

$$P(\theta|D) = \frac{\cancel{e^{\log(D,\theta)}} \cdot e^{-\frac{1}{2}(\theta-\theta_{MAP})^T\left(-\nabla^2 \log(D,\theta)\right)(\theta-\theta_{MAP})}}{\int \cancel{e^{\log(D,\theta)}} \cdot e^{-\frac{1}{2}(\theta-\theta_{MAP})^T(\ldots \ldots)} \, d\theta}$$

$$= N\left(\theta \,\middle|\, \overset{\mu,\,\Sigma}{\theta_{MAP}, \Sigma}\right)$$

$$\Sigma^{-1} = -\nabla^2 \log(D,\theta)\Big|_{\theta_{MAP}}$$

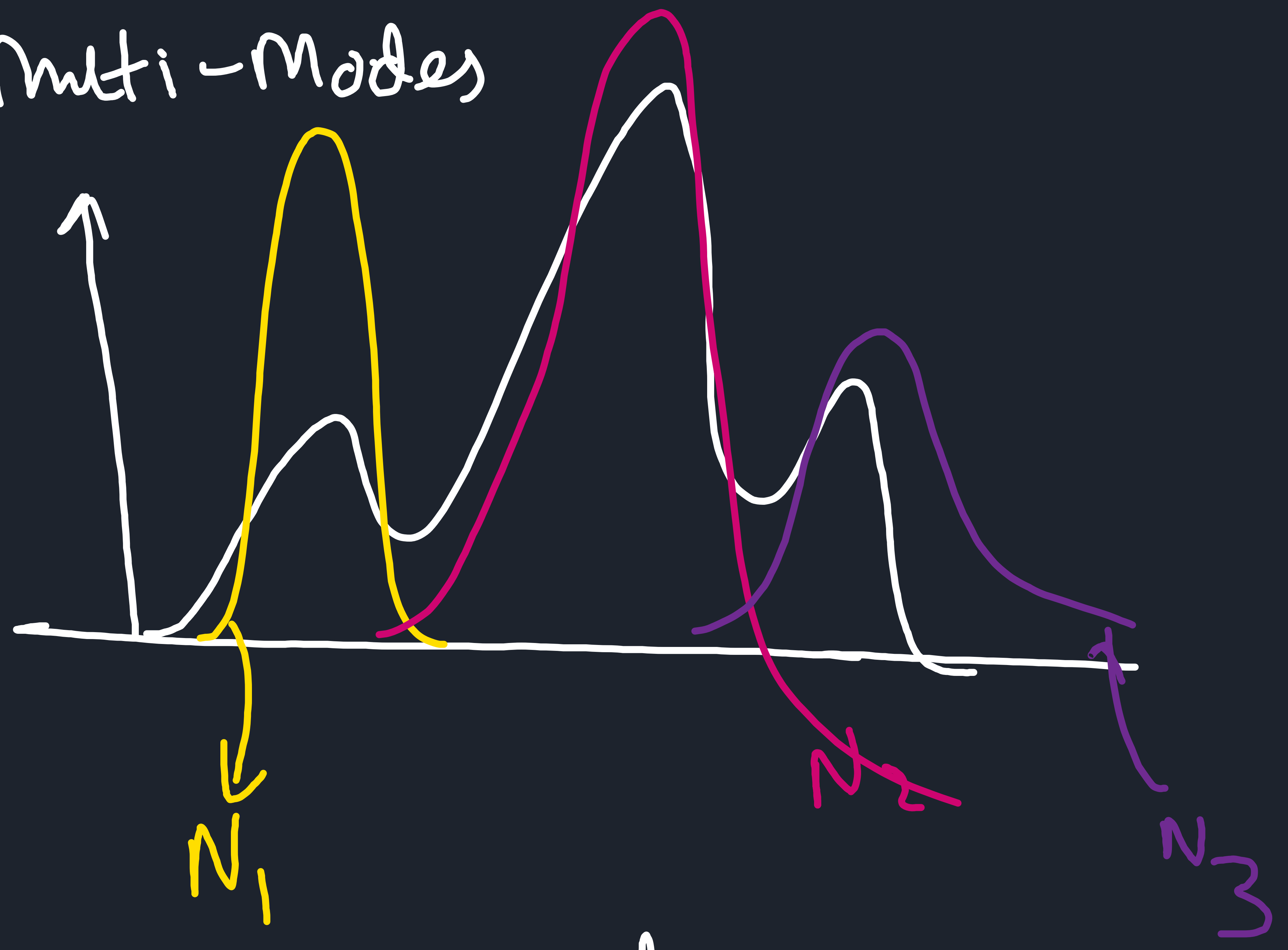
Multi Variate Normal

$$\frac{e^{-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)}}{\int e^{-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)} \, dx}$$

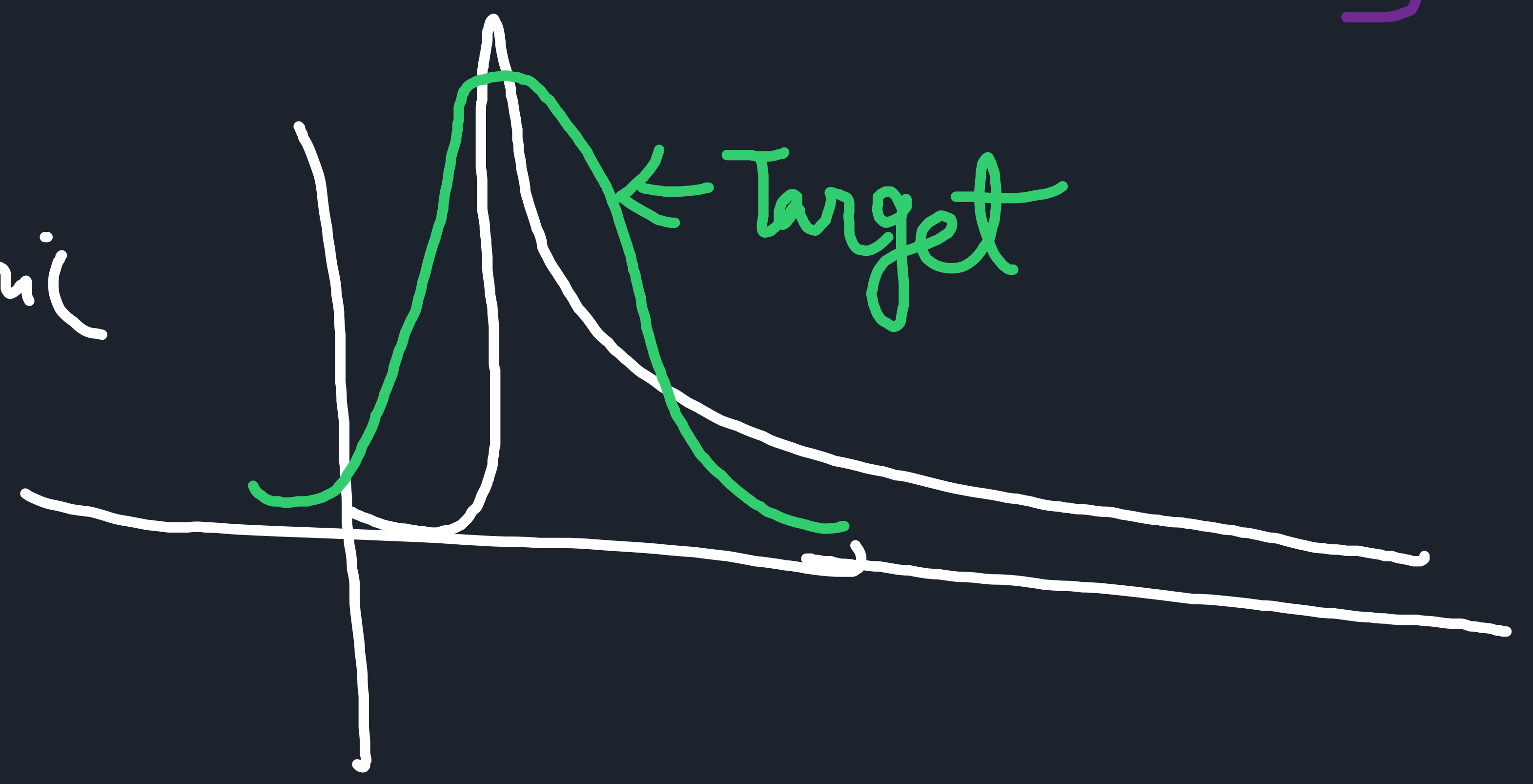
(i) Find θ_{MAP}

(ii) Find Σ of Appx. NORMAL

(i) Multi-Modes



(ii) Non-symmetric



(iii) Support Mismatch