Expectations

Expectation: The Big Idea

- Expectation (or Expected Value) is a fundamental concept in probability theory.
- It represents the long-run average value of a random variable over many trials.
- For a discrete random variable, expectation is computed as a weighted sum.

Definition

- Given a discrete random variable X with probability mass function (PMF) $P(X = x_i) = p_i$, the expectation is: E[X] = sum x_i p_i
- This sum is taken over all possible values of X.

Example: Rolling a Die

- Let X be the outcome of a fair 6-sided die.
- The possible values are $\{1, 2, 3, 4, 5, 6\}$ with equal probability $\frac{1}{6}$.
- Compute expectation:

$$E[X] = \sum_{i=1}^{6} \left(\frac{i}{6} \right)$$

Linearity of Expectation

- If X and Y are discrete random variables, then: E[aX + bY] = aE[X] + bE[Y]
- This property holds even if X and Y are dependent.
- Useful for breaking down complex expectations.

Example: Sum of Two Dice

- Let X_1 and X_2 be two independent dice rolls.
- By linearity: E[X_1 + X_2] = E[X_1] + E[X_2] = 3.5 + 3.5 = 7
- No need to compute the full distribution of the sum!

Expectation of a Function

- If $g(X)$ is a function of a discrete random variable X: E[g(X)] = sum g(x_i) p_i
- Example: If X is a fair die roll, find $E\left[X^2\right]$: E[X^2] = sum {i=1}^{6} i^2 cdot frac{1}{6}

Summary

- Expectation is the weighted sum of values of a random variable.
- Key properties:
  - Linearity: $E[aX + bY] = aE[X] + bE[Y]$
  - Function Expectation: $E[g(X)] = \sum g\left(x_i\right)p_i$
- Useful in probability, statistics, and machine learning.

Thank You!

- Questions?