Conditional Probability

A = { Covid Test + }

B = { Has Covid }

Why are A and B not same?

Conditional Probability

$A = \{ \text{Covid Test} + \}$

$B = \{ \text{Has Covid} \}$

Why are A and B not same?

Ground Truth (Actual)

Prediction/Test	NO COVID	HAS COVID
NO COVID		
HAS COVID		

Conditional Probability

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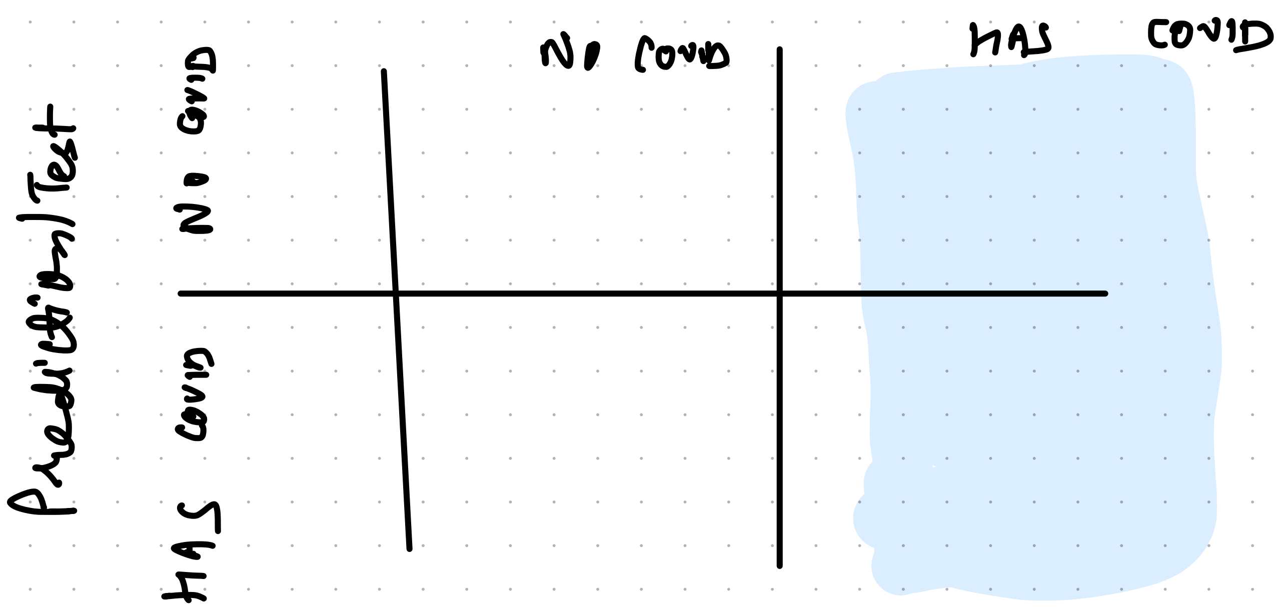
Why are A and B not same?

Ground Truth (Actual)



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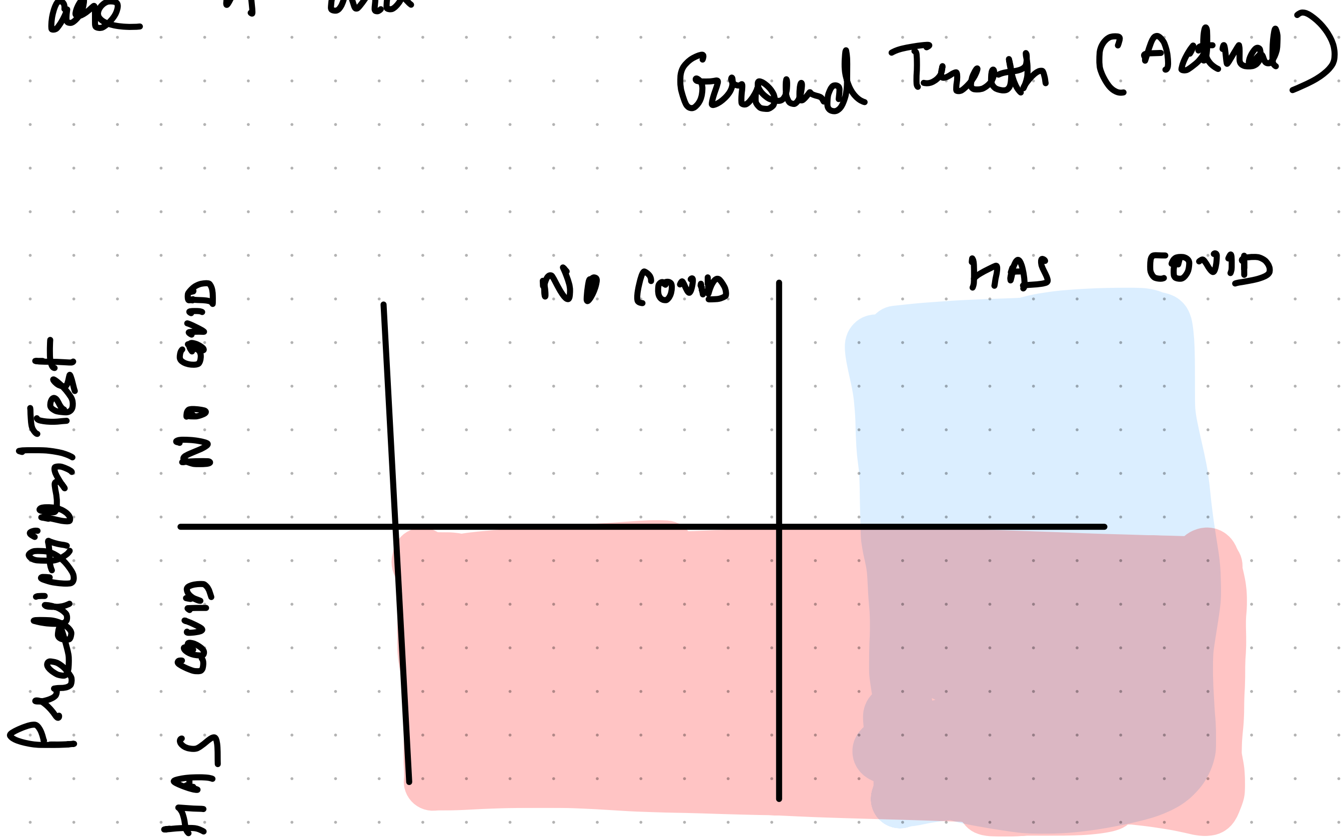

Conditional Probability

$A = \{\text{Covid Test} +\}$

$B = \{\text{Has Covid}\}$

$A = \{P_1, P_2, P_3, P_4, P_7, P_8\}$

$B = \{P_1, P_2, P_6, P_7, P_8, P_9, P_{10}\}$

$U = \{P_1 \ldots P_{12}\}$

Conditional Probability

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Confusion Matrix

		GT	
		NO COVID	COVID
Pred.	NO COVID	$A^c \cap B^c$	$A^c \cap B$
	COVID	$A \cap B^c$	$A \cap B$

Conditional Probability

$A = \{\text{Covid Test} +\}$

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$B^c = \{P_3, P_4, P_5, P_{11}, P_{12}\}$

Confusion Matrix

		GT: NO COVID	GT: COVID
Pred.	NO COVID	$A^c \cap B^c$	$A^c \cap B$
	COVID	$A \cap B^c$	$A \cap B = \{1, 2, 7, 8\}$

Conditional Probability

$A = \{\text{Covid Test} +\}$

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$A = \{P_1, P_2, P_3, P_4, P_7, P_8\}$

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$B^c = \{P_3, P_4, P_5, P_{11}, P_{12}\}$

Confusion Matrix

		G T	
		NO COVID	COVID
Pred.	NO COVID	$A^c \cap B^c = \{5, 11, 12\}$	$A^c \cap B = \{6, 9, 10\}$
	COVID	$A \cap B^c = \{3, 4\}$	$A \cap B = \{1, 2, 7, 8\}$

Conditional Probability

$A = \{\text{Covid Test} +\}$

$B = \{\text{Has Covid}\}$

$A = \{P_1, P_2, P_3, P_4, P_7, P_8\}$

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Confusion Matrix

		GT	
		NO COVID	COVID
Pred.	NO COVID	3	3
	COVID	2	4

Conditional Probability

$A = \{\text{Covid Test} +\}$

$B = \{\text{Has Covid}\}$

$A = \{P_1, P_2, P_3, P_4, P_7, P_8\}$

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Confusion Matrix

		GT: NO COVID	GT: COVID
Pred.	NO COVID	3 (TN)	3 (FN)
	COVID	2 (FP)	4 (TP)

Conditional Probability

$A = \{\text{Covid Test +}\}$

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$A = \{P_1, P_2, P_3, P_4, P_7, P_8\}$

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Ideally

TP, TN high

FN, FP low

Confusion Matrix

		GT: NO COVID	GT: COVID
Pred.	NO COVID	3 (TN)	3 (FN)
	COVID	2 (FP)	4 (TP)

Conditional Probability

$A = \{\text{Covid Test} +\}$

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$A = \{P_1, P_2, P_3, P_4, P_7, P_8\}$

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Confusion Matrix

		GT: NO COVID	GT: COVID
Pred.	NO COVID	$A^c \cap B^c$	$A^c \cap B$
	COVID	$A \cap B^c$	$A \cap B$

Conditional Probability

$A = \{ \text{Covid Test} + \}$

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$A = \{ P_1, P_2, P_3, P_4, P_7, P_8 \}$

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$U = \{ P_1 \ldots P_{12} \}$

$P[A] =$

$P[B] =$

$P[A \cap B]$

$P[A | B] =$

Conditional Probability

$A = \{ \text{Covid Test} + \}$

$B = \{ \text{Has Covid} \}$

$A = \{ P_1, P_2, P_3, P_4, P_7, P_8 \}$

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Confusion Matrix

		GT	
		NO COVID	COVID
Pred.	NO COVID	3 (TN)	3 (FN)
	COVID	2 (FP)	4 (TP)

$P[A] = 6/12 = 0.5$

$P[B] = 7/12$

$P[A \cap B] = 4/12 = 1/3$

$P[A|B] = 4/7$; $P[B|A] = 4/6 = 2/3$

Conditional Probability

$A = \{ \text{Get } 3 \}$

$B = \{ \text{odd numbers} \} = \{1, 3, 5\}$

$P[A|B] = ?$; $P[B|A] = ?$

Conditional Probability

$A = \{ \text{Get } 3 \}$

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$A \cap B = A = \{3\}$

$$P[A|B] = \frac{n(\{3\})}{n(\{1,3,5\})} = \frac{1}{3}$$

$$= \frac{P(\{3\})}{P(\{1,3,5\})} = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{3}$$

Conditional Probability

$A = \{ \text{Get } 3 \}$

$B = \{ \text{odd numbers} \} = \{1, 3, 5\}$

$P[A|B] = ?$; $P[B|A] = ?$

$A \cap B = A = \{3\}$

$$P[B|A] = \frac{n(\{3\})}{n(\{3\})} = 1$$

$$= \frac{P(\{3\})}{P(\{3\})} = \frac{\frac{1}{6}}{\frac{1}{6}} = 1$$

4 sided dice

X: first roll ; Y: second roll

B: $\min(X, Y) = 2$; $M = \max(X, Y) = 3$

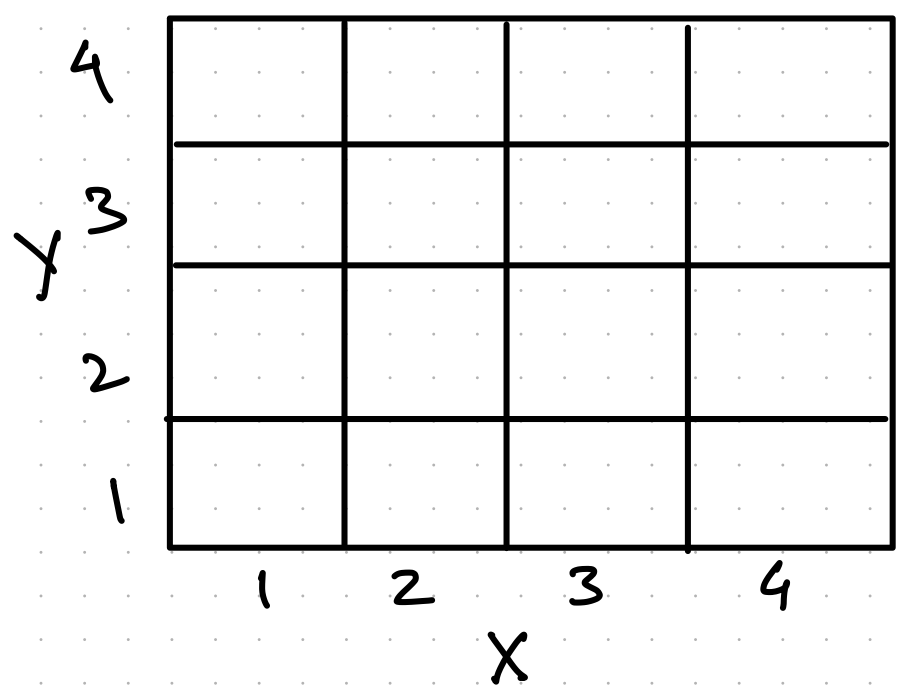
$P[M | B] = \ ?$

4 sided dice

X : first roll ; Y: second roll

B : $\min(X, Y) = 2$; $M = \max(X, Y) = 3$

$P[M \mid B] = ?$



4 sided dice

X : first roll ; Y : second roll

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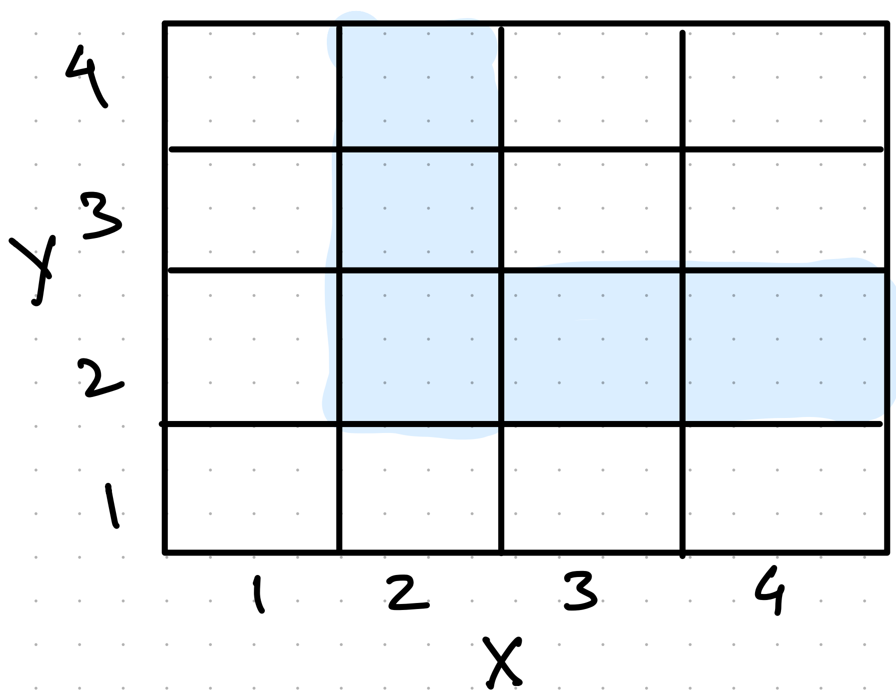


Marking B

examples

$\min(1,1) = 1 \neq 2$

$\min(2,1) = 2 = 2$

$\min(2,4) = 2 = 2$

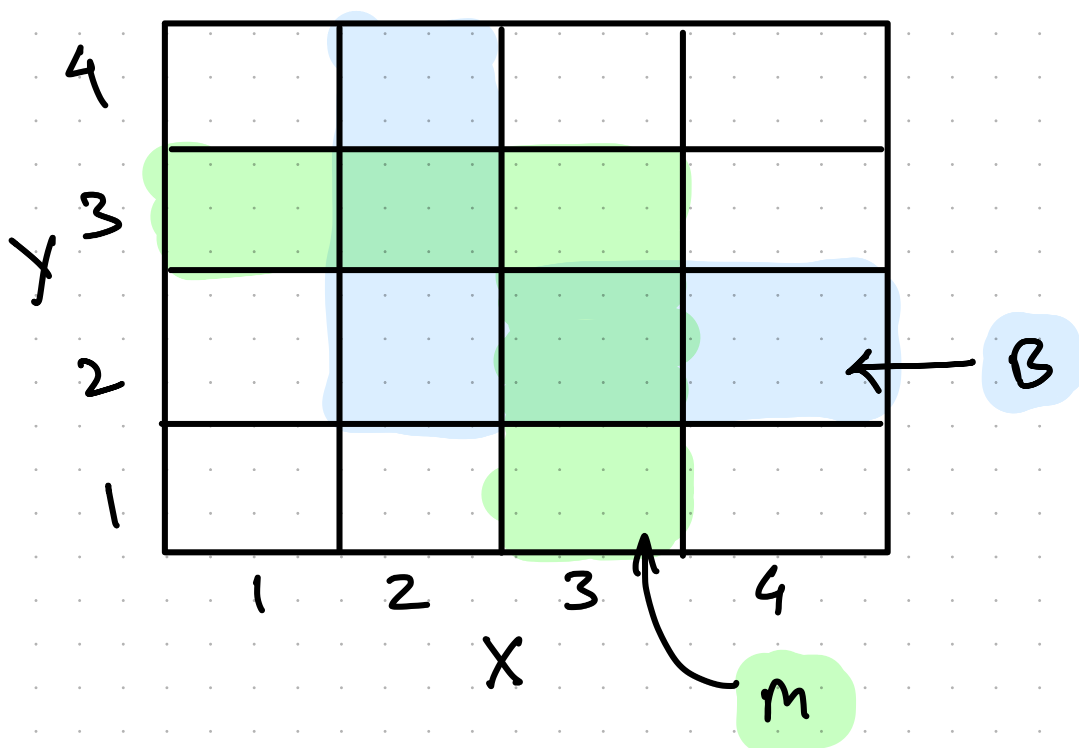
$\min(4,3) = 3 \neq 2$

4 sided dice

X : first roll ; Y : second roll

B : $\min(X, Y) = 2$; M = $\max(X, Y) = 3$

$P[M \mid B] = ?$



4 sided dice

X : first roll ; Y : second roll

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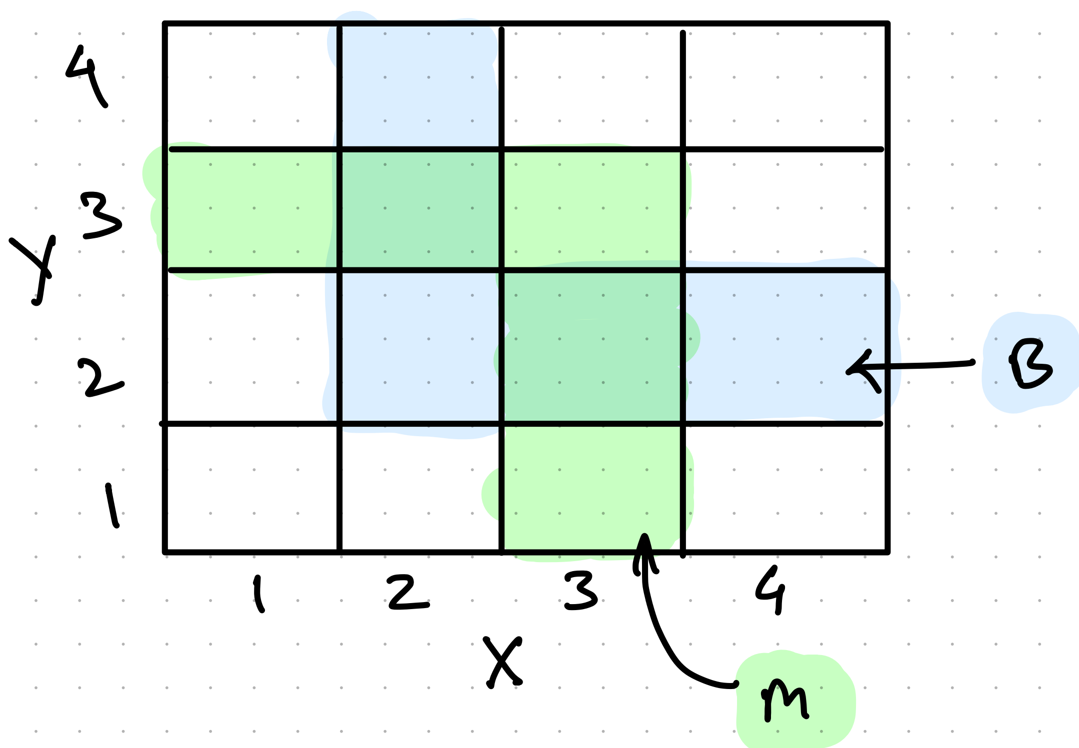
$P[M \mid B] = ?$



$n(M \cap B) = 2$

$n(B) = 5$

$\therefore P[M \mid B] = \frac{2}{5}$

Let $P[B] > 0$; $P[A|B]$ satisfies 3 Axioms of Probability

1) Non-negativity: $P[A|B] = \dfrac{P[A \cap B]}{P[B]}$ $\begin{matrix} > 0 \\ > 0 \end{matrix}$

$\therefore P[A|B] > 0$

2) Normalization: $P(\Omega|B) = 1$

Let $P[B] > 0$; $P[A|B]$ satisfies 3 Axioms of Probability

1) Non-negativity; $P[A|B] = \frac{P[A \cap B]}{P[B]} \begin{matrix} > 0 \\ > 0 \end{matrix}$

$\therefore \quad P[A|B] > 0$

2) Normalization: $P(\Omega|B) = 1$

$$= \frac{P(\Omega \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

3) Disjoint A and C

$$P[A \cup C \mid B] = P[A \mid B] + P[C \mid B]$$

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$$P[A \cup C \mid B] = P[A \mid B] + P[C \mid B]$$

LHS

$$P(A \cup C \mid B) = \frac{P[(A \cup C) \cap B]}{P[B]}$$

$$= \frac{P[(A \cap B) \cup (C \cap B)]}{P[B]}$$

3) Disjoint A and C

$$P[A \cup C \mid B] = P[A \mid B] + P[C \mid B]$$

LHS

$$P(A \cup C \mid B] = \frac{P[(A \cup C) \cap B]}{P[B]}$$

$$= \frac{P[(A \cap B) \cup (C \cap B)]}{P[B]}$$

$\because$ A and C are disjoint;

$(A \cap B)$ and $(C \cap B)$ are disjoint.

$$\therefore P[A \cup C \mid B] = \frac{P[A \cap B] + P[C \cap B]}{P[B]} = P[A \mid B] + P[C \mid B]$$

Independence

* Two coins toss
* Event $B = \{1^{st} \text{ coin is } H\}$
* Event $A = \{2^{nd} \text{ coin is } H\}$
* Does knowing "B" change anything in "A".

Independence

Two events A and B are independent if:

Knowing one occured gives no extra information about other.

$$P(A \mid B) = P(A)$$

Independence

Two events A and B are independent if:

Knowing one occured gives no extra information about other.

$$P(A \mid B) = P(A)$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = P(A)$$

$$\Rightarrow P(A \cap B) = P(A) \cdot P(B)$$

For 2 coin toss:

$B = \{2^{nd} \text{ coin is } H\}$

$P(B) = 1/2$

$A = \{1^{st} \text{ coin is } H\}$

$P(A) = 1/2$

$P(A \mid B) = ?$

For 2 coin toss:

$B = \{1^{st} \text{ coin is } H\}$

$P(B) = 1/2$

$A = \{2^{nd} \text{ coin is } H\}$

$P(A) = 1/2$

$P(A \mid B) = ?$

If 1^{st} coin is H; 2^{nd} still has $1/2$ prob. of H

$\therefore P(A \mid B) = 1/2$

$$= \frac{P(A \cap B)}{P(B)} = \frac{\dfrac{n(\{H,H\})}{n(\{(H,H),(H,T),(T,H),(T,T)\})}}{1/2}$$

$$= \frac{1}{2}$$

For 2 coin toss:

$B = \{1^{st} \text{ coin is } H\}$

$P(B) = 1/2$

$A = \{2^{nd} \text{ coin is } H\}$

$P(A) = 1/2$

$\Omega = \{HH, HT, TH, TT\}$

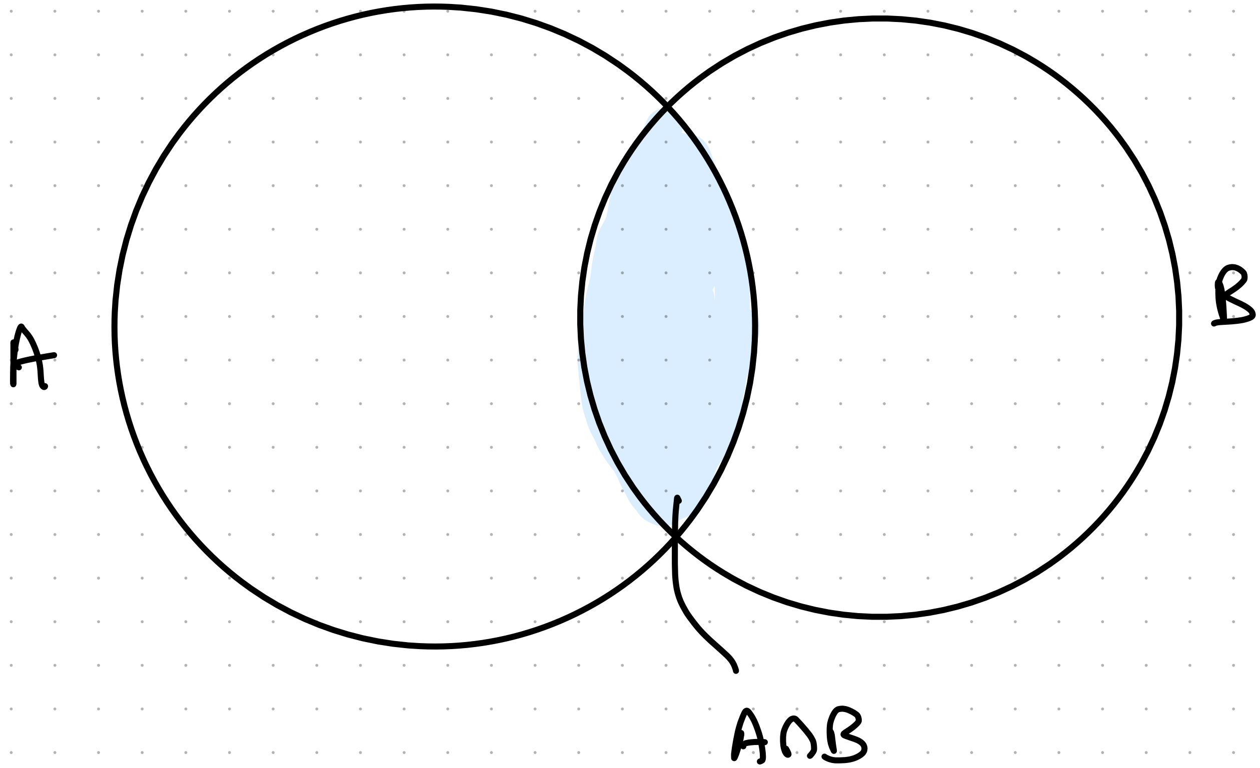
$P(A|B) = \frac{1}{2} = \frac{}{}$

Disjoint v/s Independent.

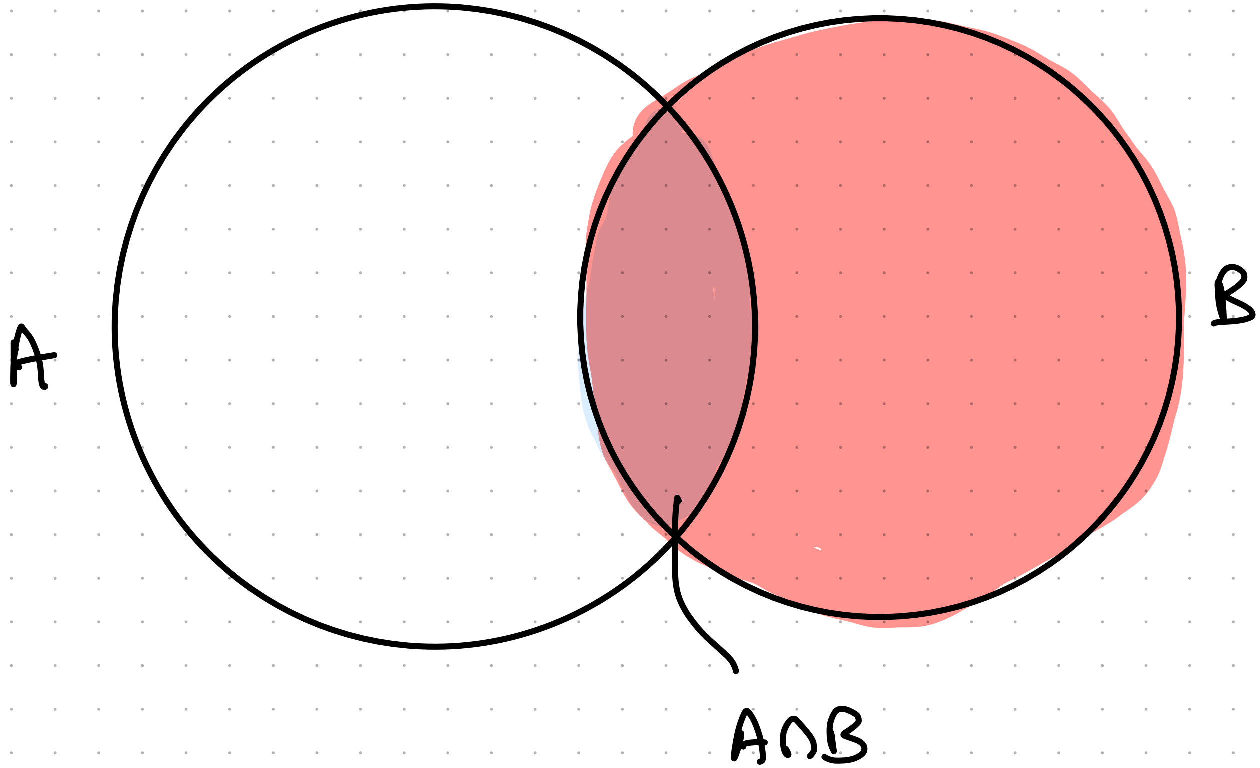
Disjoint $\quad A \cap B = \phi$

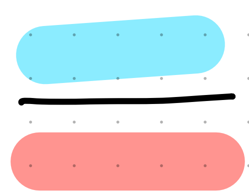
$\Rightarrow \quad P(A \cap B) = 0$

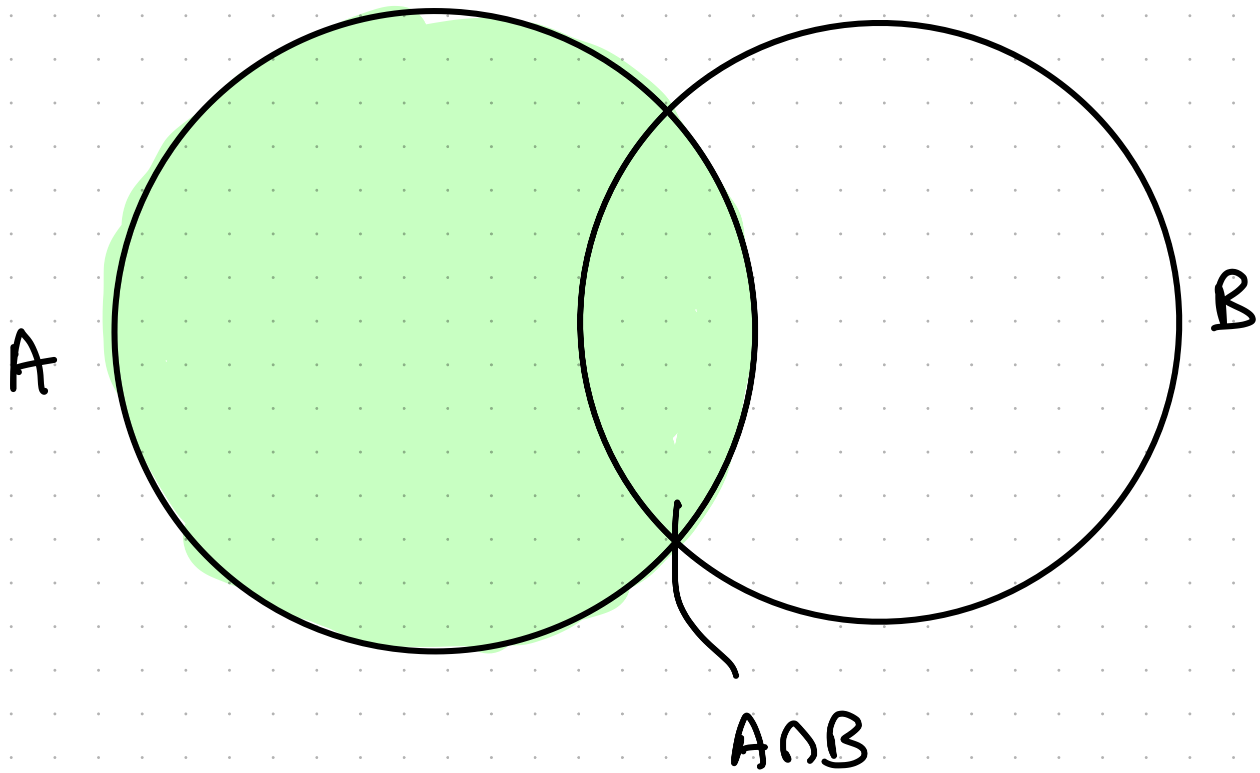
Independent $\quad P(A \cap B) = P(A) \cdot P(B)$



$P(A|B) =$

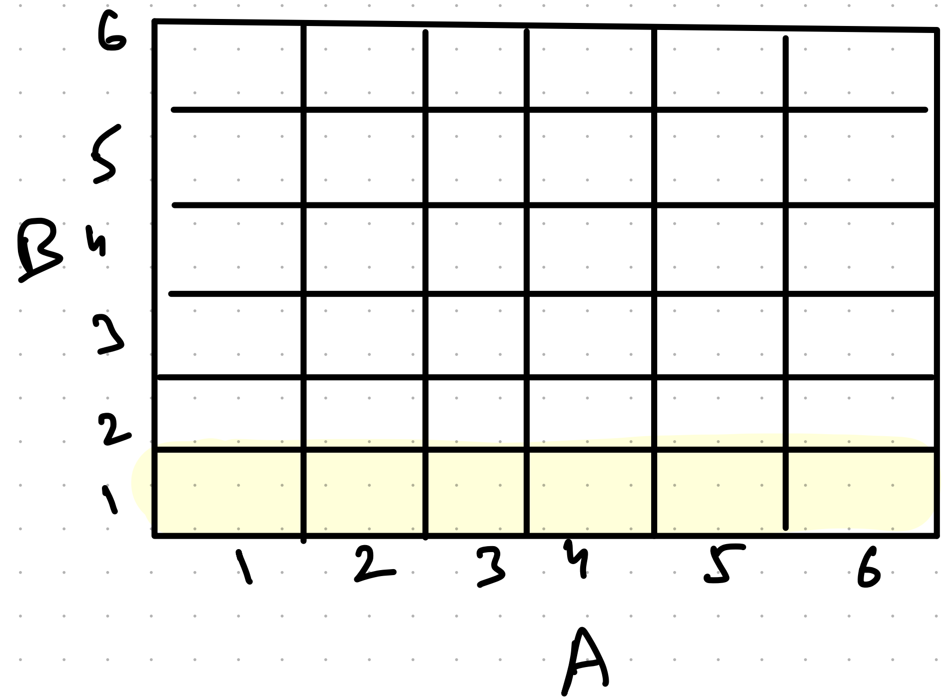


$P(A|B) =$ 



$$P(A|B) = \frac{\qquad}{\qquad} = \frac{P(A)}{P(\Omega)} = \frac{\qquad}{1}$$

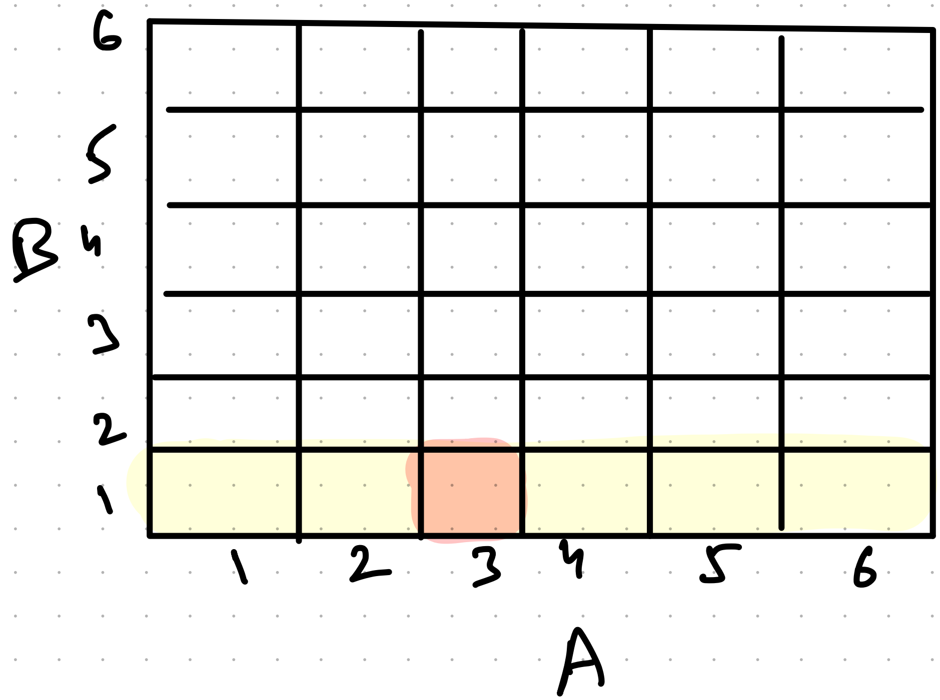
I) $A = \{1^{st} \text{ dice is } 3\}$; $B = \{2^{nd} \text{ dice is } 4\}$



$P(A) = \frac{1}{6} =$ ______

$P(B) = \frac{1}{6}$

I) $A = \{1^{st}$ dice is $3\}$; $B = \{2^{nd}$ dice is $4\}$



$P(A) = \frac{1}{6} = \frac{\quad}{\quad}$

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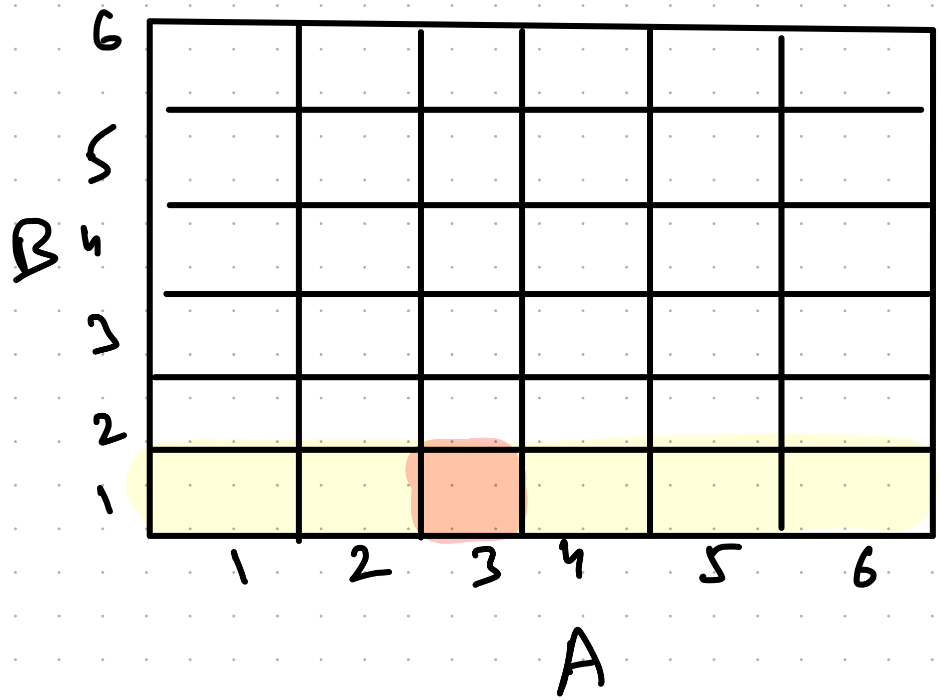
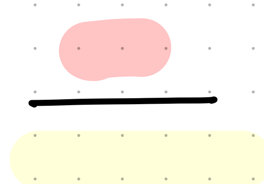


$P(A) = \frac{1}{6} =$ 

$P(B) = \frac{1}{6}$

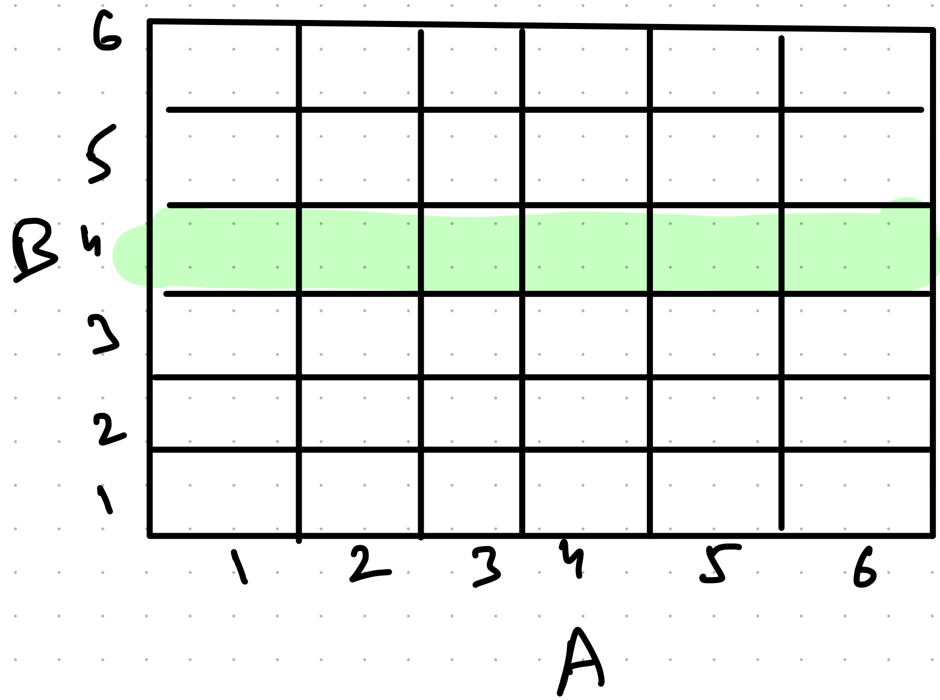
$P(A \cap B) = \frac{1}{36}$

$P(A) \cdot P(B) = \frac{1}{36}$

$= P(A \cap B)$

$\therefore$ A & B are independent.

I) $A = \{1^{st}$ dice is $3\}$; $B = \{2^{nd}$ dice is $4\}$



$$P(A|B) = \frac{1}{6} = P(A)$$

II) $A = \{ 1^{st} \text{ dice is } 1 \}$

$B = \{ \text{sum of first \& second is } 7 \}$

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$B = \{ \text{sum of first \& second is } 7 \}$

$$P(A) = \frac{1}{6}$$

$$B = \{ (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) \}$$

$$P(B) = \frac{6}{36} = \frac{1}{6}$$

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$$P(B) = \frac{6}{36} = \frac{1}{6}$$

$A \cap B = \{(1,6)\}$; $P(A \cap B) = \frac{1}{36}$

$P(A) \cdot P(B) = \frac{1}{36}$ $\therefore$ A & B are independent

II) $A = \{ 1^{st} \text{ dice is } 1 \}$

$B = \{ \text{sum of first \& second is } 7 \}$

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$$P(B) = \frac{6}{36} = \frac{1}{6}$$

$$P(A|B) = \frac{n(\{(1,6)\})}{n(B)} = \frac{1}{6} = P(A)$$

II) $A = \{1^{st} \text{ dice is } 1\}$

$B = \{\text{sum is } 8\}$

$P(A) = \frac{1}{6}$

$B = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$

If B occurs, can A occur?

$P(A|B) = 0$

$\neq P(A)$

$\therefore$ A & B are dependent

IV)

$A = \{ 1^{st} \text{ dice is } 2 \}$

$B = \{ \text{sum is } 8 \}$

$B = \{ (2,6), (3,5), (4,4), (5,3), (6,2) \}$

$P(B) = \frac{5}{36}$; $P(A) = \frac{1}{6}$

$$P(A|B) = \frac{n(\{(2,6)\})}{n(B)} = \frac{1}{5} \neq P(A)$$

$\therefore$ A & B are dependent

V) $A = \{\text{max is } 2\}$; $B = \{\text{min is } 2\}$

$$A = \{(1,2), (2,1), (2,2)\}$$

$$B = \{(2,2), (2,3), (2,4), (2,5), (2,6), (3,2), (4,2), (5,2), (6,2)\}$$

$$A \cap B = \{(2,2)\}$$

$$P(A) = \frac{3}{36} = \frac{1}{12} \; ; \quad P(A|B) = \frac{n(A \cap B)}{n(B)} = \frac{1}{9} \neq P(A)$$

i. i. d.

Sample #	Colour	Radius	Condition
	0.9	0.7	
1			
2	0.6	0.	
3			
4			
⋮			

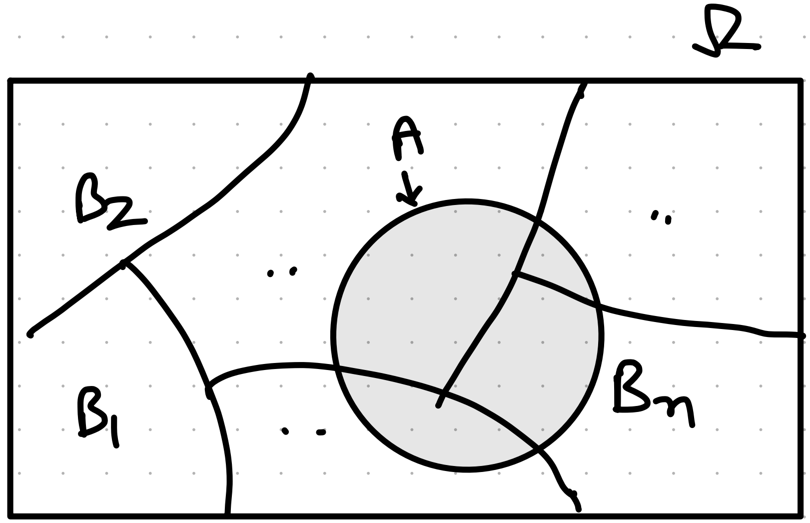
Total Probability Law

$\{B_1, \ldots B_n\}$ is partition on Ω

(i) $B_i \cap B_j = \phi$ if $i \neq j$

(ii) $\bigcup_{i=1}^{n} B_i = \Omega$

Total Probability Law



$\{B_1, \ldots B_n\}$ is partition on Ω

(i) $B_i \cap B_j = \phi$ if $i \neq j$

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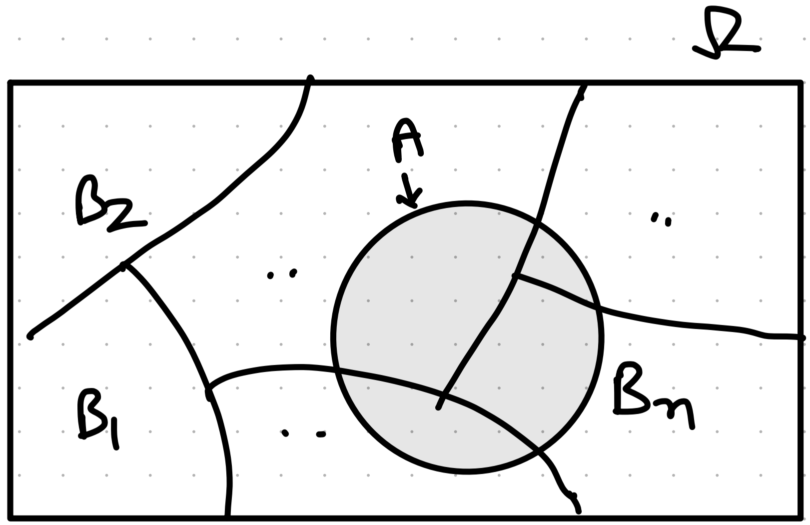
$$P(A) = \sum_{i=1}^{n} P(A \mid B_i) \cdot P(B_i)$$

Total Probability Law



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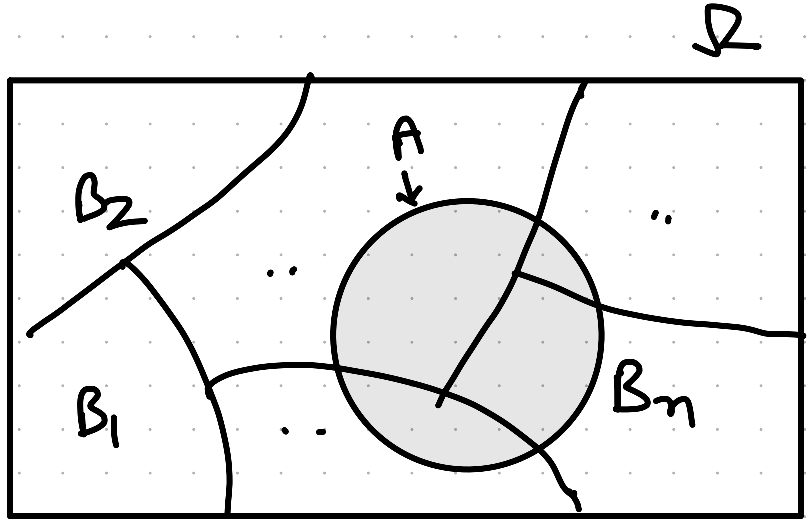
$$P(A) = \sum_{i=1}^{n} P(A|B_i) \cdot P(B_i)$$

Prove

Total Probability Law



$\{B_1, \ldots B_n\}$ is partition on Ω

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(ii) $\bigcup_{i=1}^{n} B_i = \Omega$

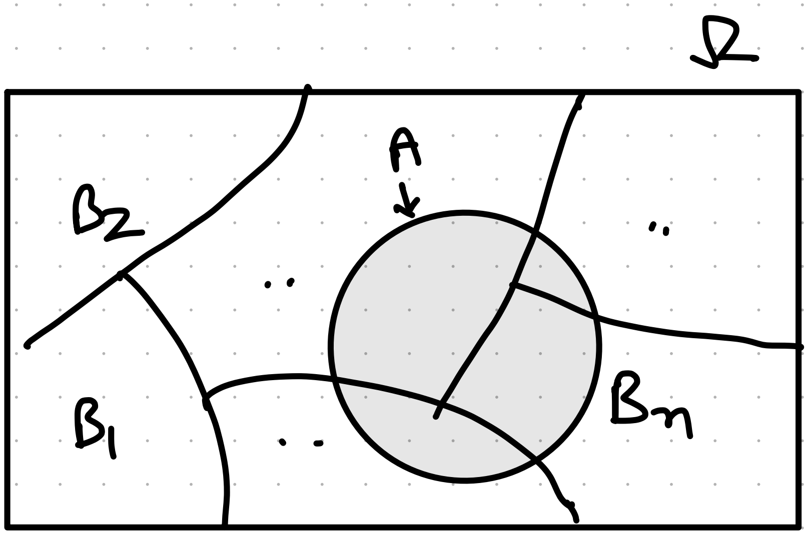
$$P(A) = \sum_{i=1}^{n} P(A \mid B_i) \cdot P(B_i)$$

Prove

$$RHS = \sum_{i=1}^{n} P(A \cap B_i) = P[(A \cap B_1) \cup (A \cap B_2) \ldots (A \cap B_n)]$$

$$= P[A]$$

Total Probability Law



$\{B_1, \dots B_n\}$ is partition on Ω

(i) $B_i \cap B_j = \phi$ if $i \neq j$

(ii) $\bigcup_{i=1}^{n} B_i = \Omega$

We can also write

$$P(A) = P(A \mid B) \cdot P(B) + P(A \mid B^c) \cdot P(B^c)$$

Q) COVID example

Sensitivity = TRUE POSITIVE RATE = $P(\text{Test}^+ \mid \text{COVID}) = 0.90$

Specificity = TRUE negative Rate = $P(\text{Test}^- \mid \neg \text{COVID}) = 0.95$

Probability that Random person has COVID = 0.05

What is $P(\text{Test}^+)$

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What is $P(\text{Test}^+)$

$P(\text{covid}) = 0.05$; $P(\neg \text{covid}) = 0.95$

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What is $P(\text{Test}^+)$

$P(\text{Covid}) = 0.05$; $P(\neg \text{Covid}) = 0.95$

$$P(\text{Test}^+) = P(\text{Test}^+ \mid \text{covid}) \cdot P(\text{Covid}) + P(\text{Test}^+ \mid \neg \text{covid}) \cdot P(\neg \text{Covid})$$

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$$P(\text{Test}^+) = P(\text{Test}^+ \mid \text{covid}) \cdot P(\text{Covid}) + P(\text{Test}^+ \mid \neg \text{covid}) \cdot P(\neg \text{Covid})$$

$$= 0.9 * .05 + (1 - 0.95) * .95$$

$$= 0.045 + .0475 = 0.0925$$

Q) Flight delays

$P(S) = 0.5$; $P(C) = 0.3$; $P(R) = 0.2$

Sun Cloud Rain

$P(D|S) = 0.1$; $P(D|C) = 0.3$; $P(D|R) = 0.6$

$P(D) = ?$ (Any Random day flight delay)

Q) Flight delays

$P(S) = 0.5$; $P(C) = 0.3$; $P(R) = 0.2$

Sun Cloud Rain

$P(D|S) = 0.1$; $P(D|C) = 0.3$; $P(D|R) = 0.6$

$P(D) = ?$ (Any Random day flight delay)

$$P(D) = P(D|S) \cdot P(S) + P(D|R) \cdot P(R) + P(D|C) \cdot P(C)$$

$$= .5 * .1 + .6 * .2 + .3 * .3$$

$$= .26$$

Q) COVID example

Sensitivity = TRUE POSITIVE RATE = $P(\text{Test}^+ \mid \text{covid}) = 0{\cdot}90$

Specificity = TRUE negative Rate = $P(\text{Test}^- \mid \neg\text{covid}) = 0{\cdot}95$

Probability that Random person has COVID = 0·05

What is $P(\text{Test}^+)$

$P(\text{Covid}) = 0{\cdot}05$; $P(\neg\,\text{Covid}) = 0{\cdot}95$

$P(\text{Test}^+) = 0{\cdot}0925$

$$\boxed{P(\text{COVID} \mid \text{Test}^+) = \ ?}$$

Bayes Theorem

$$P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}$$

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$$P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}$$

$$P(\text{COVID} \mid \text{Test}^+) = \frac{P(\text{Test}^+ \mid \text{COVID}) \cdot P(\text{COVID})}{P(\text{Test}^+)}$$

Bayes Theorem

$$P(B \mid A) = \frac{P(A \mid B) \cdot P(B)}{P(A)}$$

$$P(\text{COVID} \mid \text{Test}^+) = \frac{P(\text{Test}^+ \mid \text{COVID}) \cdot P(\text{COVID})}{P(\text{Test}^+)}$$

$P(\text{COVID})$ = Prior probability of covid w/o Test

$P(\text{COVID} \mid \text{Test}^+)$ = Posterior probability after Test (updated belief)

Bayes Theorem

$$P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}$$

$$P(\text{COVID} \mid \text{Test}^+) = \frac{P(\text{Test}^+ \mid \text{COVID}) \cdot P(\text{COVID})}{P(\text{Test}^+)}$$

Sensitivity = TRUE POSITIVE RATE = $P(\text{Test}^+ \mid \text{COVID}) = 0.90$

Specificity = TRUE negative Rate = $P(\text{Test}^- \mid \neg \text{COVID}) = 0.95$

Probability that Random person has COVID = 0.05

$P(\text{Test}^+) = .09725$

Bayes Theorem

$$P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}$$

$$P(\text{COVID} \mid \text{Test}^+) = \frac{P(\text{Test}^+ \mid \text{COVID}) \cdot P(\text{COVID})}{P(\text{Test}^+)}$$

Sensitivity = TRUE POSITIVE RATE = $P(\text{Test}^+ \mid \text{covid}) = 0.90$

Specificity = TRUE negative Rate = $P(\text{Test}^- \mid \neg\text{covid}) = 0.95$

Probability that Random person has COVID = 0.05

$P(\text{Test}^+) = .09725$

$$P(\text{COVID} \mid \text{Test}^+) = \frac{.90 * .05}{.09725} = 0.486$$

Bayes Theorem

$$P(\text{COVID} \mid \text{Test}^+) = \frac{P(\text{Test}^+ \mid \text{COVID}) \cdot P(\text{COVID})}{P(\text{Test}^+)}$$

Sensitivity = TRUE POSITIVE RATE = $P(\text{Test}^+ \mid \text{COVID}) = 0.90$

Specificity = TRUE negative Rate = $P(\text{Test}^- \mid \neg \text{COVID}) = 0.95$

$$P(\text{COVID} \mid \text{Test}^+) = \frac{.90 \times .05}{.09725} = 0.486$$

(App)

Monty Hall Problem

3 doors

- One door has a car behind it
- Goat behind other two
- You initially choose a door
- Host (Monty Hall) opens one of remaining door that has a goat
- Question
  - Stay with your door
  - Switch

Monty Hall Problem

- Say you chose Door 1.

- Define

  C_1 : Car behind Door 1

  C_2 : " " Door 2

  C_3 : " " Door 3

Monty Hall Problem

- Say you chose Door 1.

- Define

  C_1 : Car behind Door 1

  C_2 : " " Door 2

  C_3 : " " Door 3

  $$P(C_1) = P(C_2) = P(C_3) = \frac{1}{3}$$

Monty Hall Problem

- Say you chose Door 1.

- Define

  C_1 : Car behind Door 1

  C_2 : " " Door 2

  C_3 : " " Door 3

- Hall opens
  - Door 1 $\to H_1$

    2 $\to H_2$

    3 $\to H_3$

- Hall never opens door with car

Monty Hall Problem

- You picked Door 1
- Case I
    - Car behind Door 1
    - Hall chooses D_2 or D_3 with equal prob
    - $\Rightarrow P(H_2 | C_1) = P(H_3 | C_1) = 0.5$

Monty Hall Problem

- You picked Door 1

- <u>Case I</u>
  - Car behind Door 1
  - Hall chooses D_2 or D_3 with equal prob
  - $\Rightarrow P(H_2 \mid C_1) = P(H_3 \mid C_1) = 0.5$

- <u>Case II</u>
  - Car behind Door 2
  - Hall has to open Door 3
  - $\Rightarrow P(H_2 \mid C_2) = 0$; $P(H_3 \mid C_2) = 1$

Monty Hall Problem

- You picked Door 1

- <u>Case I</u>
  - Car behind Door 1
  - Hall chooses D_2 or D_3 with equal prob
  - $\Rightarrow P(H_2 \mid C_1) = P(H_3 \mid C_1) = 0.5$

- <u>Case II</u>
  - Car behind Door 2
  - Hall has to open Door 3
  - $\Rightarrow P(H_2 \mid C_2) = 0$; $P(H_3 \mid C_2) = 1$

- <u>Case III</u>
  - Car behind Door 3
  - $P(H_2 \mid C_3) = 1$; $P(H_3 \mid C_3) = 0$

Monty Hall Problem

- You picked Door 1
- Let's assume Hall opened Door 3

Monty Hall Problem

- You picked Door 1
- Let's assume Hall opened Door 3
- Question

$$P(C_2 \mid H_3) = ? \quad \text{(Switch)}$$

Monty Hall Problem

- You picked Door 1
- Let's assume Hall opened Door 3
- Question

$P(C_2 | H_3) = ?$ (Switch)

$P(C_1 | H_3) = ?$ (stay)

$P(C_1) = \frac{1}{3}$ (No data on door)

Monty Hall Problem

$$P(C_2 \mid H_3) = ? \quad (\text{Switch})$$

$$P(C_2 \mid H_3) = \frac{P(H_3 \mid C_2) \cdot P(C_2)}{P(H_3)}$$

Now $P(H_3) =$

Monty Hall Problem

$$P(C_2 | H_3) = ? \quad \text{(Switch)}$$

$$P(C_2 | H_3) = \frac{P(H_3 | C_2) \cdot P(C_2)}{P(H_3)}$$

Now $P(H_3) = P(H_3 | C_1) \cdot P(C_1) + P(H_3 | C_2) \cdot P(C_2) + P(H_3 | C_3) \cdot P(C_3)$

$$= 0.5 \times \frac{1}{3} + 1 * \frac{1}{3} + 0 \times \frac{1}{3}$$

$$= \frac{1}{2} \times \frac{1}{3} + 1 * \frac{1}{3} = \frac{3}{2} \times \frac{1}{3} = \frac{1}{2}$$

Monty Hall Problem

$$P(C_2 | H_3) = ? \quad (\text{Switch})$$

$$P(C_2 | H_3) = \frac{P(H_3 | C_2) \cdot P(C_2)}{P(H_3)}$$

$$P(H_3) = 1/2$$

$$P(C_2 | H_3) = \frac{1 \times \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3} = 66.7\%$$

Monty Hall Problem

$$P(C_2 | H_3) = ? \quad \text{(Switch)}$$

$$P(C_2 | H_3) = \frac{P(H_3 | C_2) \cdot P(C_2)}{P(H_3)}$$

$$P(H_3) = 1/2$$

$$P(C_2 | H_3) = \frac{1 \times \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3} = 66.7\%$$

$$P(C_1 | H_3) = \frac{1}{3} = 33.3\%$$

Monty Hall Problem

$$P(C_1) = 33.3\%$$

$$P(C_2 | H_3) = 66.7\%$$

$$P(C_1 | H_3) = 33.3\%$$

$\therefore$ Presented w/ new data,
switching to Door 2 is better