Discrete

PMF $p_X(x)$

$$E[X] = \sum_x x\, p_X(x)$$

Continuous

PDF $f_X(x)$

$$E[X] = \int x\, f_X(x)\, dx$$

1) Uniform Distribution

$$f_X(x) = \begin{cases} \dfrac{1}{b-a} & ; \quad a \le x \le b \\ 0 & ; \quad o/w \end{cases}$$

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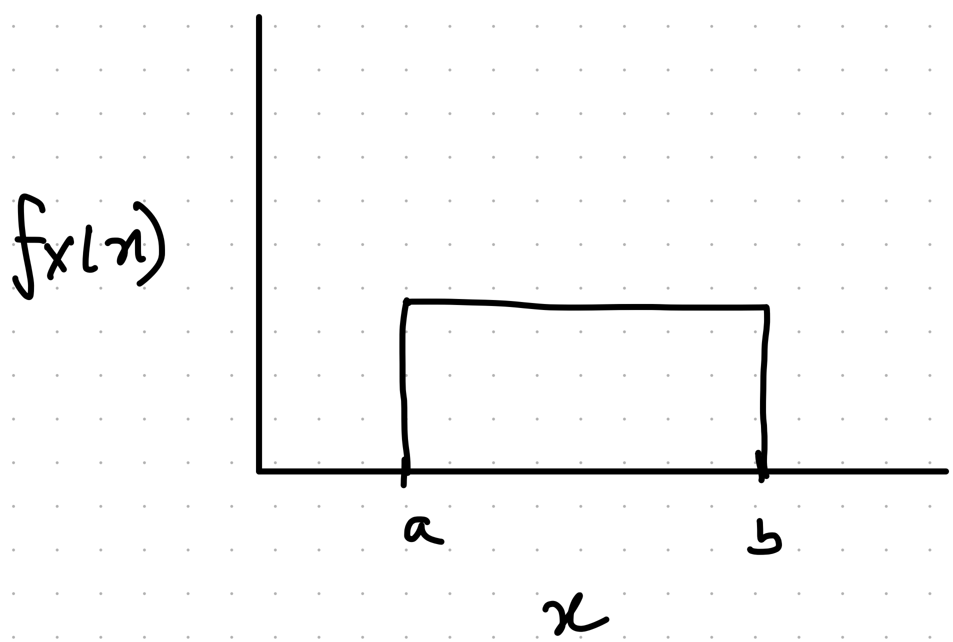

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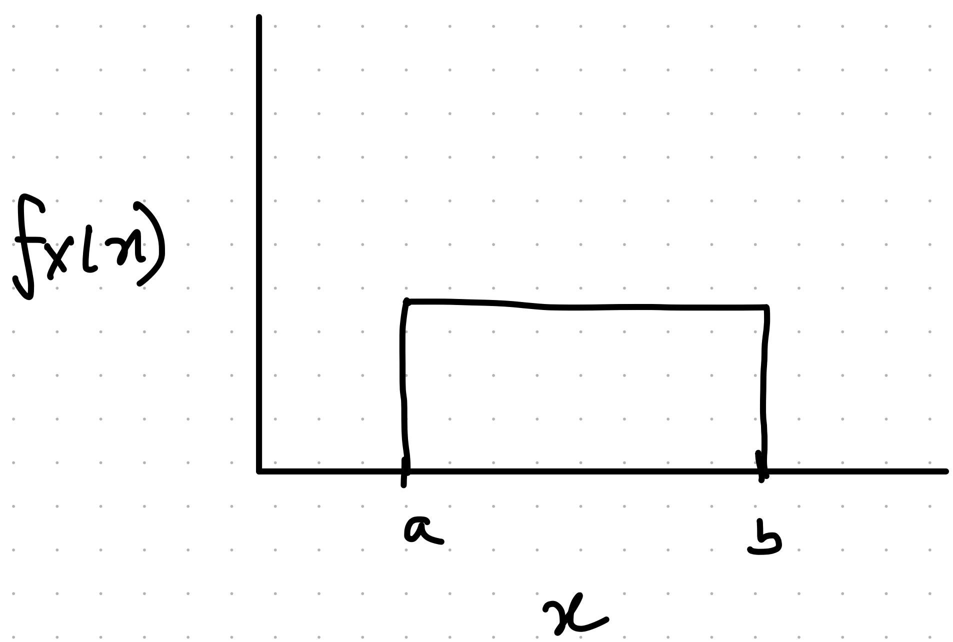


$$E[X] = \int_{-\infty}^{\infty} f_X(x) \cdot x \, dx$$

$$= \int_{a}^{b} f_X(x) \cdot x \, dx$$

$$= \frac{1}{b-a} \cdot \frac{x^2}{2} \Big|_a^b$$

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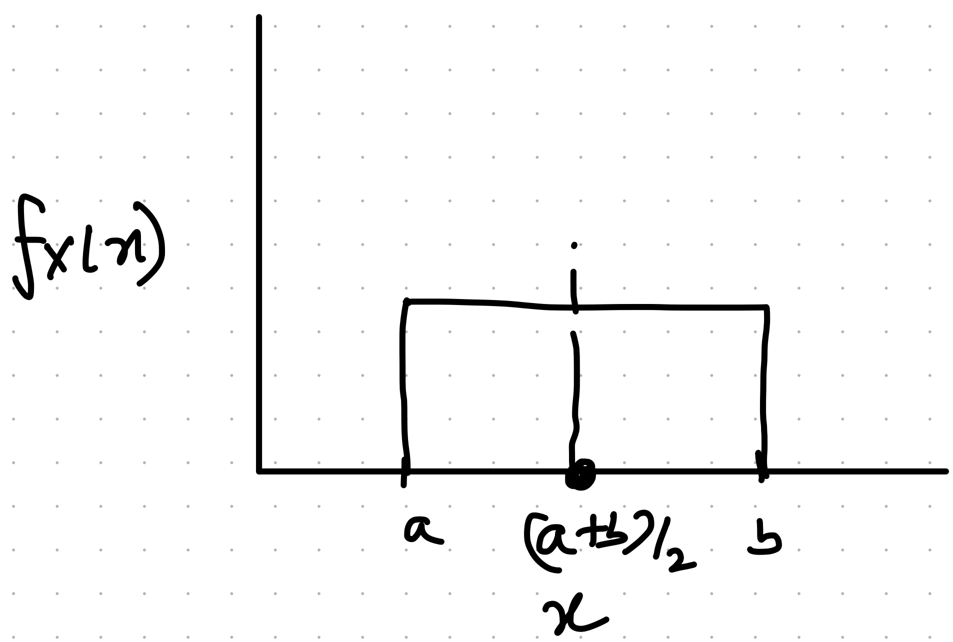

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$$E[X] = -x e^{-\lambda x}\Big|_0^\infty - \int_0^\infty (-e^{-\lambda x}) \cdot dx$$

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$$= \frac{-1 e^{-\lambda x}}{\lambda}\Big|_0^\infty = -\frac{1}{\lambda}\left[e^{-\infty} - e^{0}\right] = \frac{1}{\lambda}$$

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$$E[X^2] = \int_a^b \frac{x^2}{b-a}\, dx$$

$$= \frac{x^3}{3(b-a)} \Big|_a^b = \frac{b^3 - a^3}{3(b-a)}$$

$$= \frac{(b-a)(a^2 + b^2 + ab)}{3(b-a)}$$

Q) $VAR[X]$

$X \sim UNIF(a, b)$

$$VAR[X] = E[X^2] - E[X]^2$$

$$= \frac{a^2 + b^2 + ab}{3} - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{4a^2 + 4b^2 + 4ab - 3a^2 - 3b^2 - 6ab}{12}$$

$$= \frac{a^2 + b^2 - 2ab}{12} = \frac{(a-b)^2}{12}$$

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let $y = x - \mu$

$dy = dx$

$$E[X] = \int_{-\infty}^{\infty} (y+\mu) \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{\frac{-y^2}{2\sigma^2}}$$

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