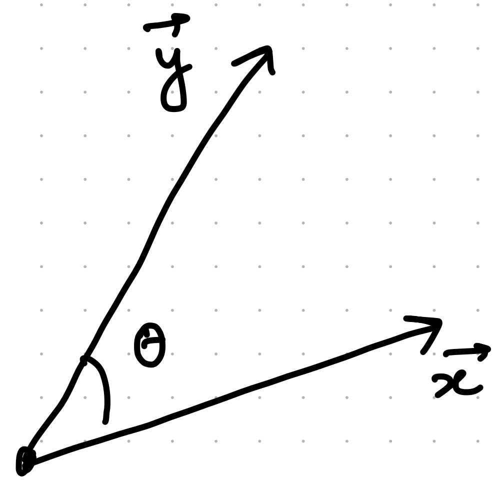
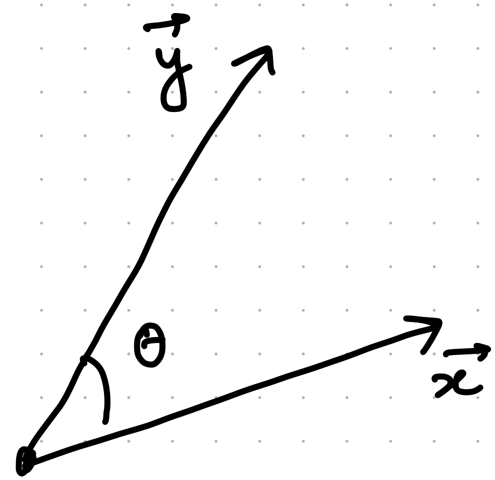


$$\cos\theta = \frac{\vec{x} \cdot \vec{y}}{\|\vec{x}\|\|\vec{y}\|} = \frac{x^T y}{\|x\| \; \|y\|}$$

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( Notebook : gensim , vision )

Consider dataset

| Id | weight (x) | Height (y) |
| --- | --- | --- |
| .. | | |
| .. | | |
| .. | | |
| .. | | |
| .. | | |
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## Refresher

1D case

say $X = [x_1 \; \ldots \; x_N]^T$

$$\bar{X} = \text{Sample mean} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

Now

$$E[X] = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} x_i$$

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$$\boxed{\cos\theta = \frac{x^T y}{\|x\| \, \|y\|} = \frac{E[XY]}{\sqrt{E[x^2]}\sqrt{E[Y^2]}}}$$

Assuming $\infty$ length vectors

Q) $E[XY] = \lim_{n\to\infty} \sum_{i=1}^{n} x_i y_i$ | $E[XY] = \sum_{y} \sum_{x} xy \, p_{X,Y}(x,y)$

why single summation in one and double in other.

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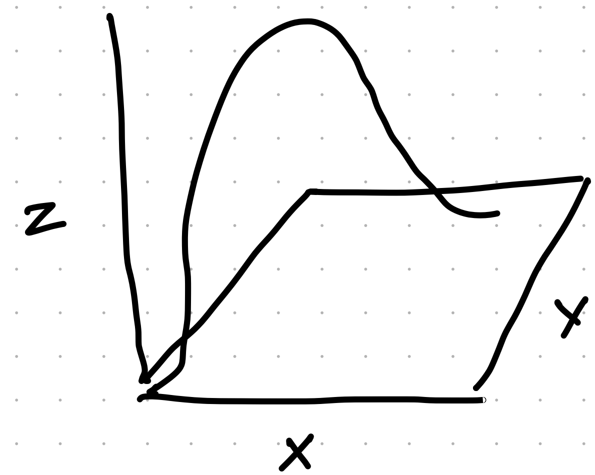

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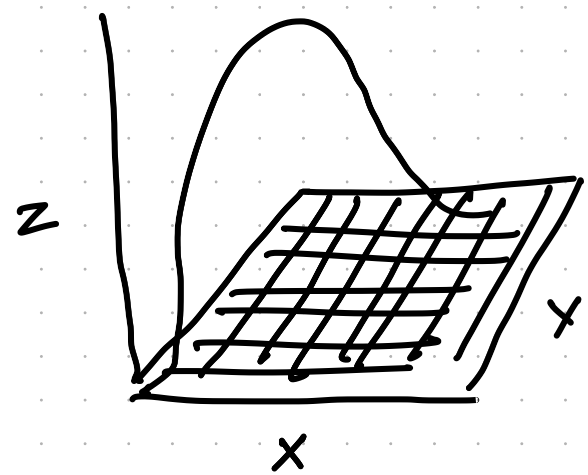

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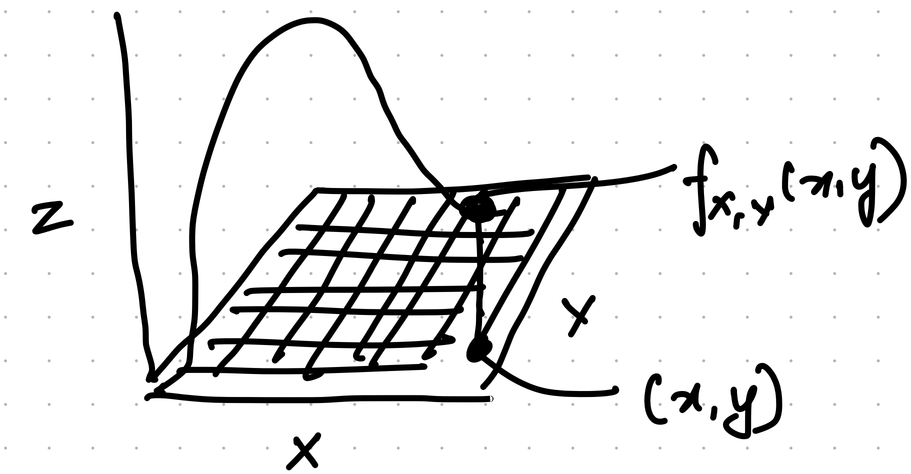

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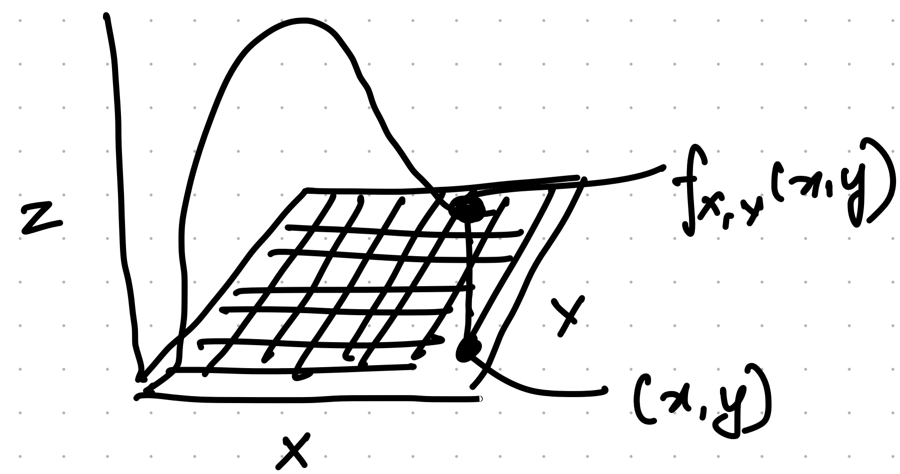


weigh over all $(x,y)$ by $f_{X,Y}(x,y)$ or $p_{X,Y}(x,y)$

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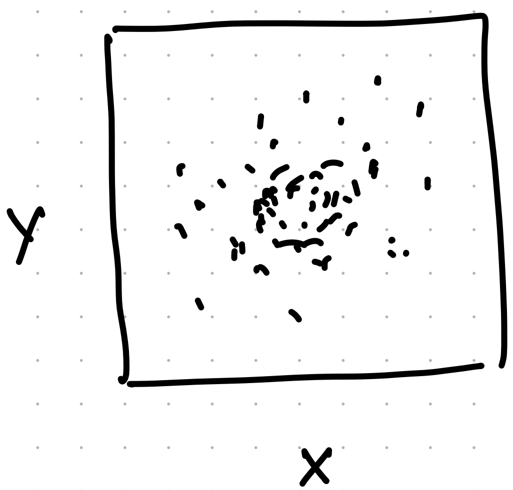
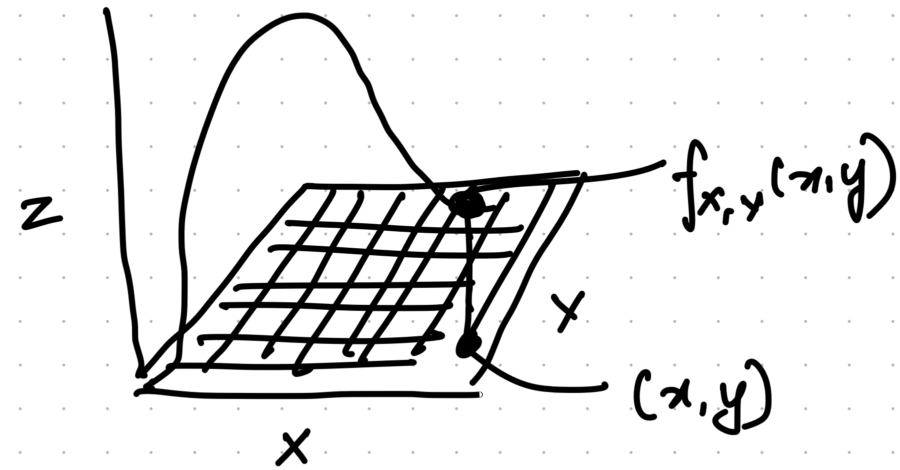



Samples are distributed as per $f_{X,Y}$




weigh over all $(x,y)$ by $f_{X,Y}(x,y)$ or $p_{X,Y}(x,y)$

Uncorrelated $\not\Rightarrow$ Independence

$Cov(x, y) = 0$

Say $X \sim U(-1, 1)$

$Y = X^2$

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Uncorrelated $\not\Rightarrow$ Independence

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Say $X \sim U(-1,1)$

$$Y = X^2$$

$$E[XY] = E[X^3] = \int_{-1}^{1} x^3\, dx$$

$$= \frac{x^4}{4}\Big|_{-1}^{1} = \frac{1-1}{4} = 0$$

Uncorrelated $\not\Rightarrow$ Independence

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But clearly $Y = X^2$ $\therefore$ $Y$ is dependent on $X$