How to model rainfall? (which distribution)

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Q: What is the probability of rain today?

- X = { RAIN, NORAIN }

$P(\text{Rain}) = .3$

$P(\text{No Rain}) = .7$

$X \sim \text{Bernoulli}(0.3)$

How to model rainfall? (which distribution)

Q: What is the probability of 5 times rain in next 10 days

$X \sim \text{Binomial}(N = 10; p = .3)$

How to model rainfall? (which distribution)

Q: What is rain category tomorrow?

	P
No Rain (0 mm)	.3
Light (0-5mm)	.4
Moderate (5-15 mm)	.2
Heavy (15+ mm)	.1

$$X \sim \text{Categorical}([.3, .4, .2, .1])$$

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Q: What is the number of raindrops hitting per minute.

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$\lambda = 10$ drops/min = Avg.

$X \sim \text{Poisson}(\lambda = 10)$

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$P(X = 3\,mm) = ?$

$P(X = 3.1415926535\,mm) = ?$

How to model rainfall? (which distribution)

Q: What is the exact rainfall (in mm)?

X is a continuous random variable

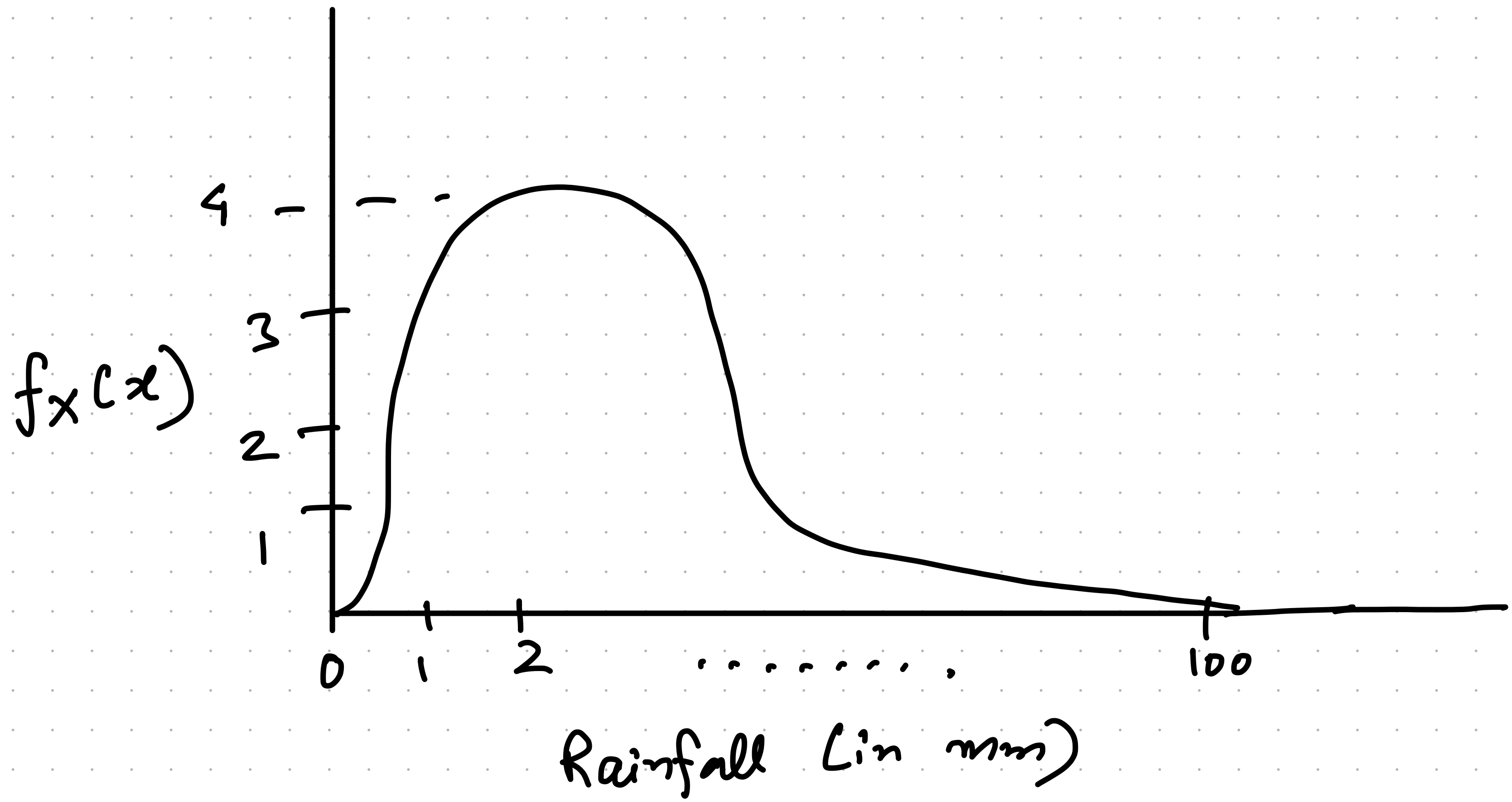
$$P(X = 3\,mm) = 0$$

$$P(X = 3.14159\,26535\,mm) = 0$$

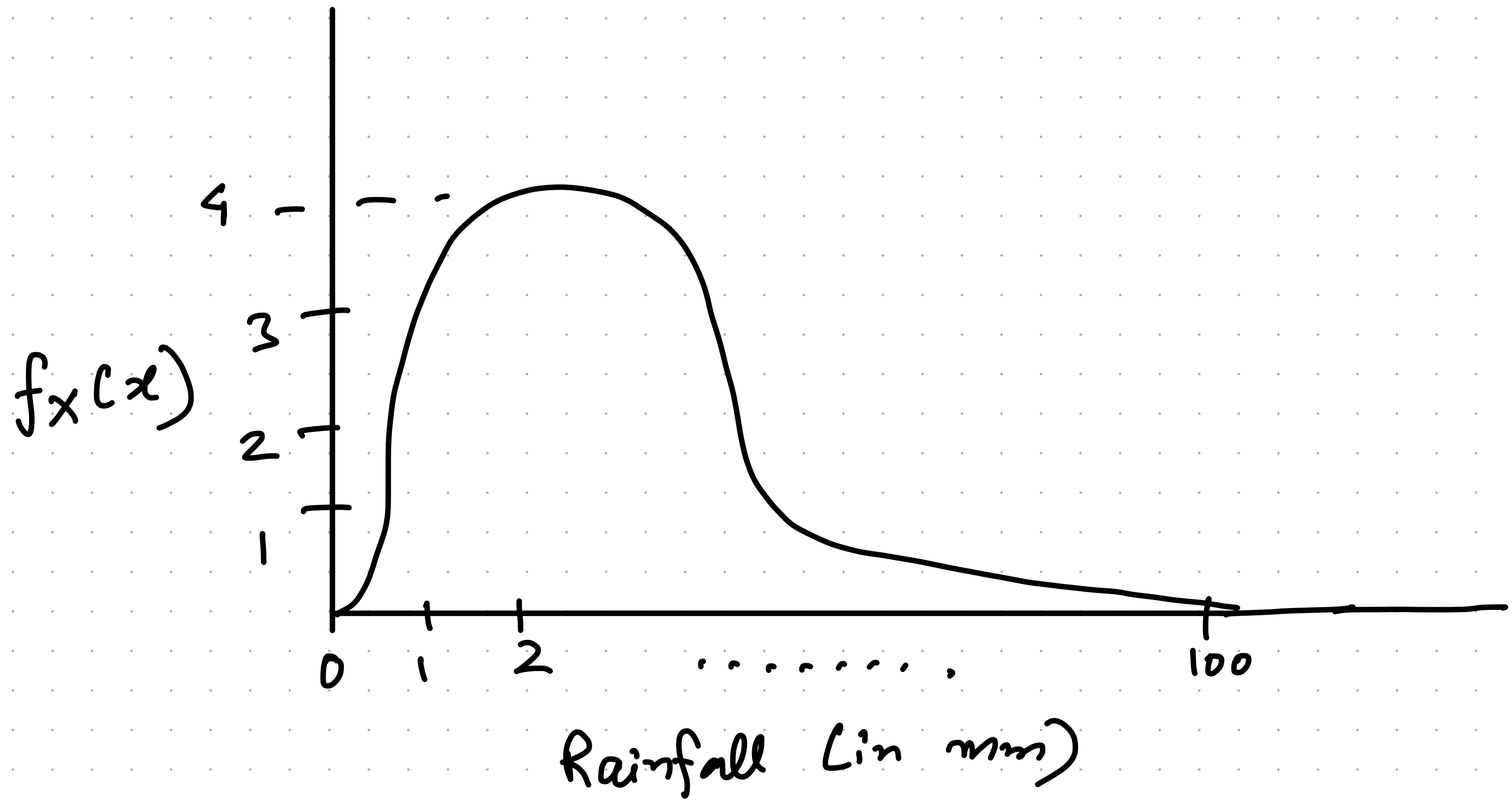
Better question: $P(2\,mm \le X \le 4\,mm) \neq 0$

For continuous r.v., we define probability of a range instead of point

Probability Density Function



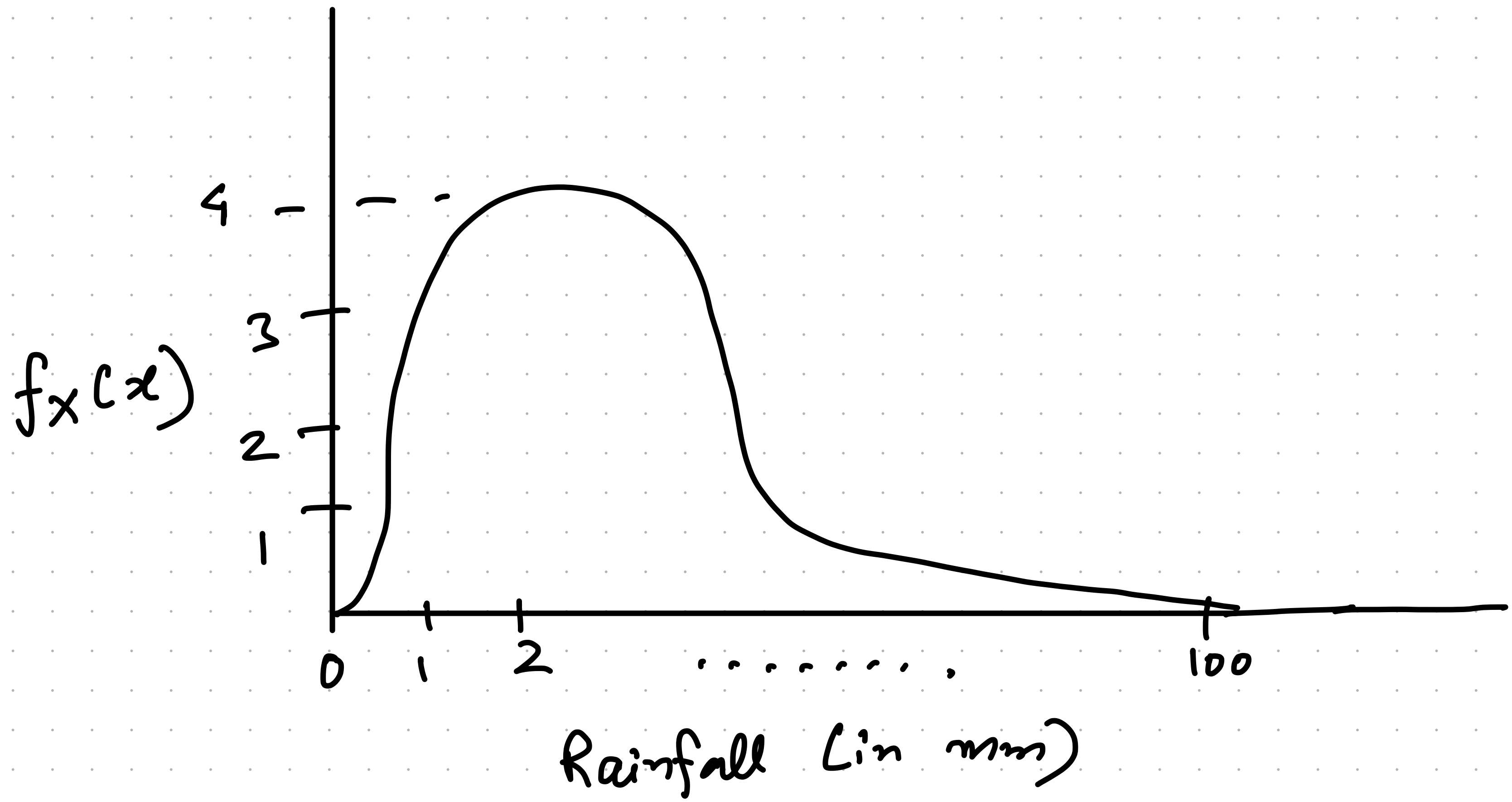
Probability Density Function



Probability density function of r.v. X :

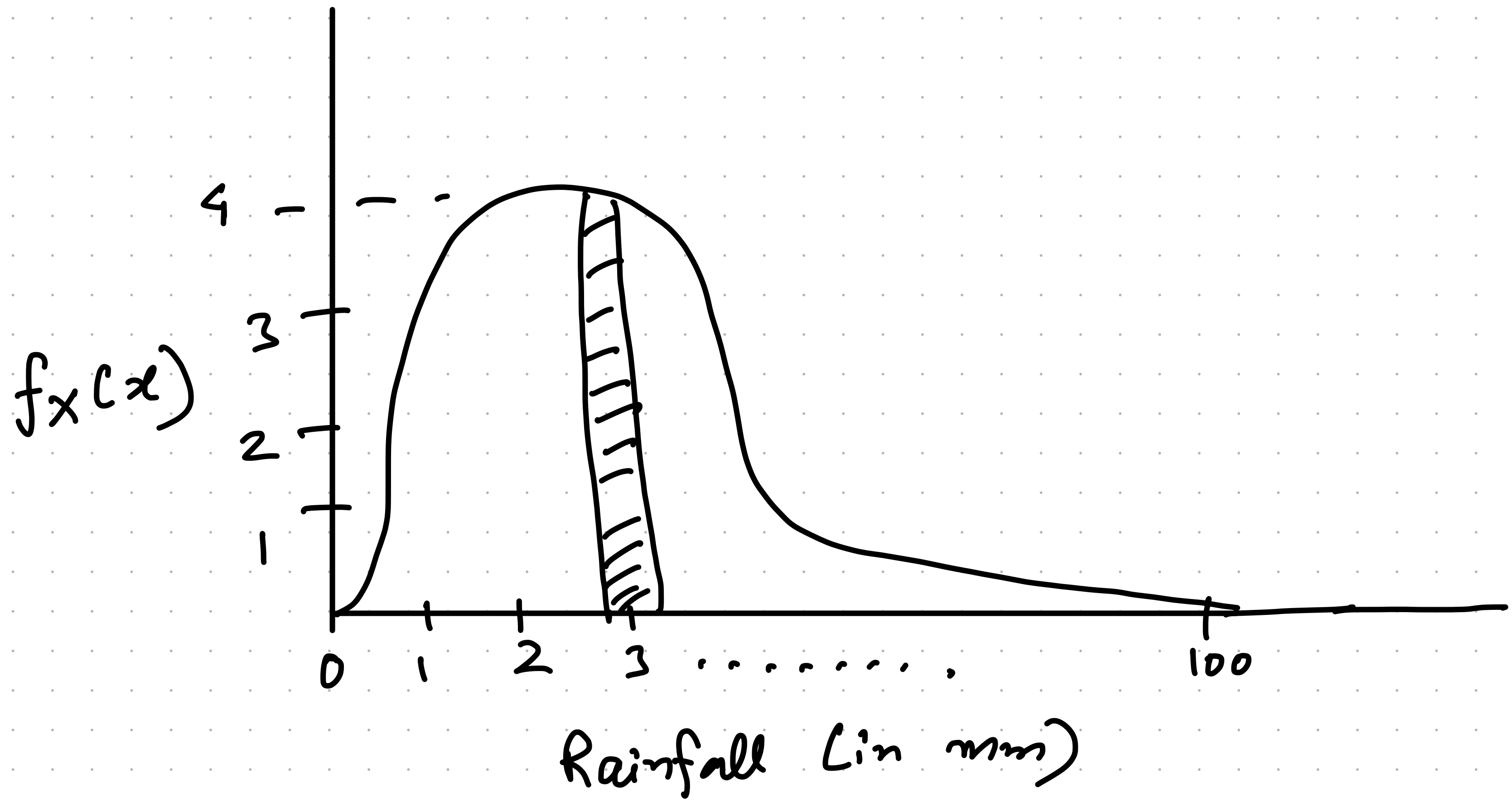
$f_X(x)$ = Probability per unit length

Probability Density Function



A = Event that Rainfall b/w 2.9 mm and 3.1 mm

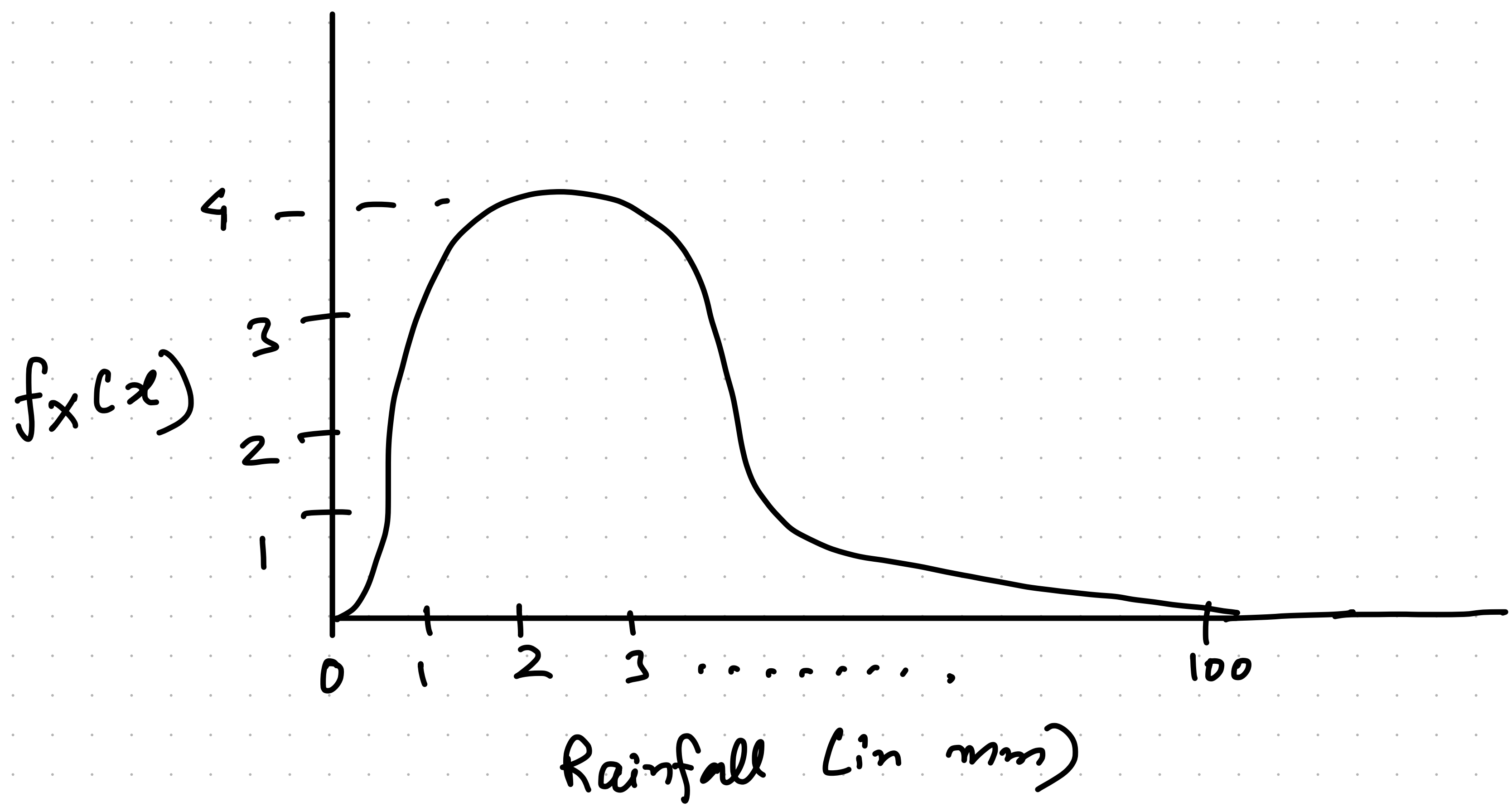
Probability Density Function



A = Event that Rainfall b/w 2.9 mm and 3.1 mm

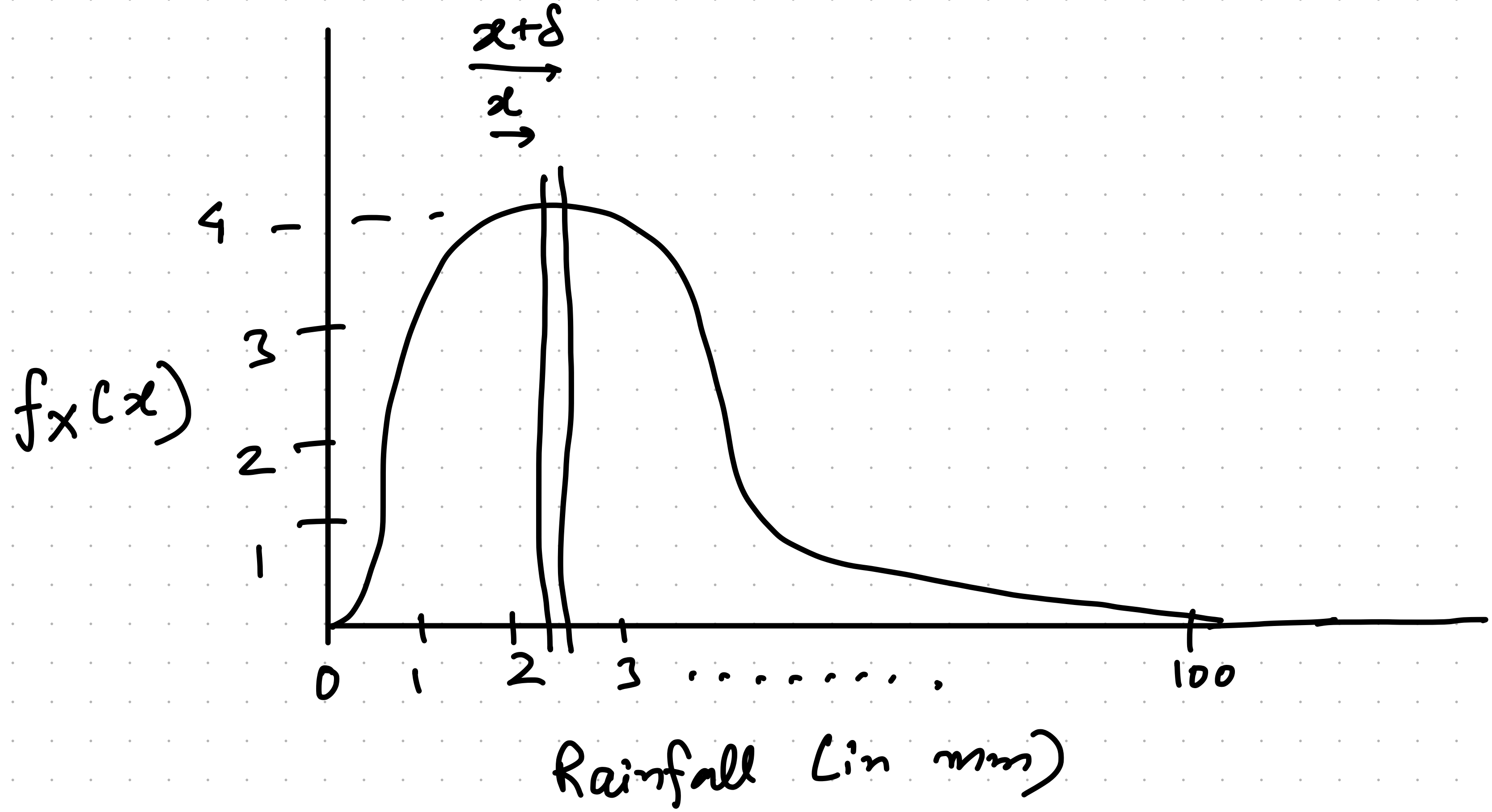
$$P[A] = \int_{x=2.9}^{3.1} f_X(x)\,.\,dx \quad = \text{Area under } f_X \text{ from } 2.9 \text{ to } 3.1$$

Probability Density Function



How can $f_X(x) > 1$??

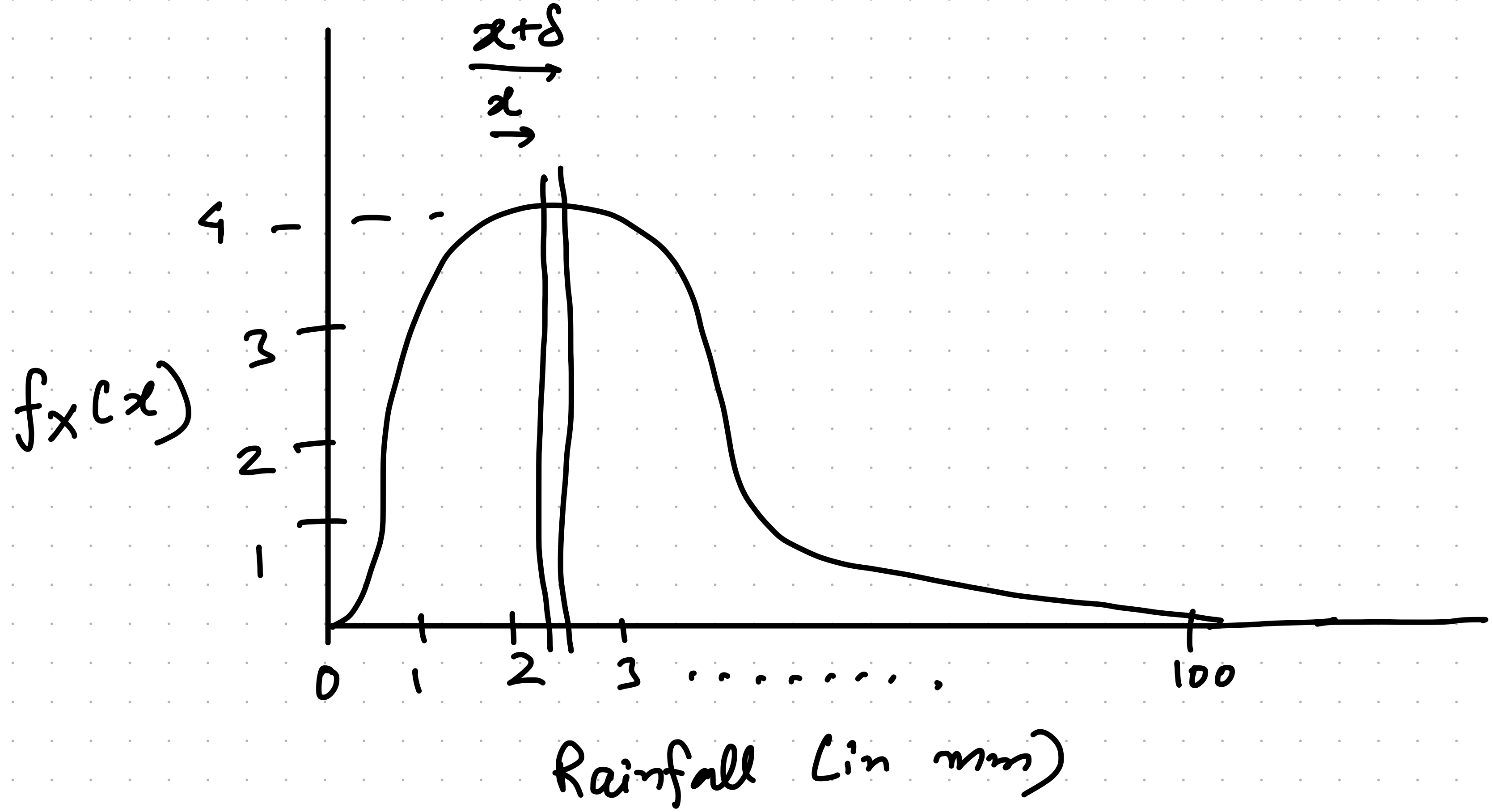
Probability Density Function



How can $f_X(x) > 1$??

$$P[x \le X \le x+\delta] = \int_{x}^{x+\delta} f_X(x)\,dx \approx f_X(x)\cdot\delta$$

Probability Density Function



How can $f_X(x) > 1$??

$$P[x \le X \le x+\delta] \approx f_X(x) \cdot \delta$$

This is b/w 0 and 1 $\Rightarrow f_X(x) \cdot \delta \le 1$

with small δ ; $f_X(x)$ can be large