$2^4 = 16$

1	2	3	4	
F	F	F	F	( )
F	F	F	T	(4)
F	F	T	F	(3)
			⋮	

Probability Laws

① $P(A) \geq 0 \quad \forall A \in \mathcal{F}$

② $P(\Omega) = 1$

③ For disjoint $[A_1, \dots A_n]$; $P\left[\bigcup_{i=1}^{N} A_i\right] = \sum_{i=1}^{N} P[A_i]$

Q) Phone $P[A^c] = 1 - P[A]$

Q) Prove $P[A^c] = 1 - P[A]$

$$A \cup A^c = \Omega$$

$$P[A \cup A^c] = P[\Omega] = 1 = P[A] + P[A^c]$$

$$\Rightarrow \quad P[A^c] = 1 - P[A]$$

Q) Prove for any $A \subseteq \Omega$; $P[A] \leq 1$

Q) Prove for any $A \subseteq \Omega$; $P[A] \leq 1$

$$P[A] + P[A^c] = 1$$

$$P[A^c] \geq 0$$

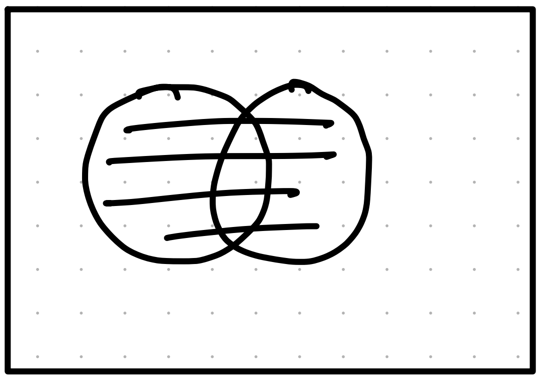
$$\Rightarrow P[A] \leq 1$$

For any A and B

$$P[A \cup B] = P[A] + P[B] - P[A \cap B]$$

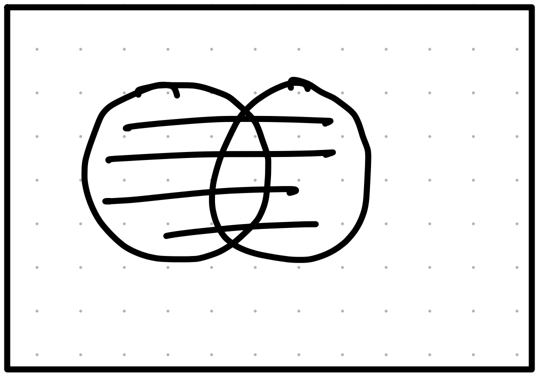
For any A and B

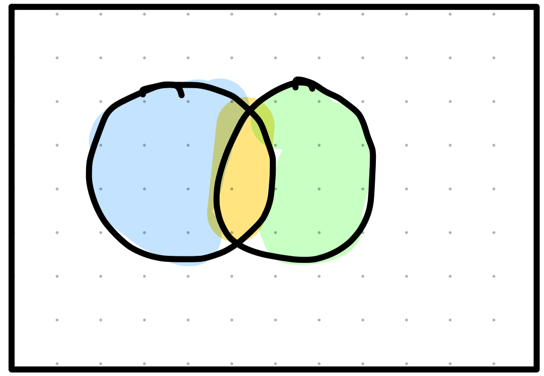
$$P[A \cup B] = P[A] + P[B] - P[A \cap B]$$



For any A and B

$$P[A \cup B] = P[A] + P[B] - P[A \cap B]$$





$$= P[A \setminus B] + P[A \cap B] + P[B \setminus A]$$

For any A and B

$$P[A \setminus B] + P[A \cap B] + P[B \setminus A]$$

$$= P[A \cap B^c] + P[A \cap B] + P[B \cap A^c]$$

For any A and B

$$P[A\setminus B] + P[A\cap B] + P[B\setminus A]$$

$$= P[A\cap B^c] + P[A\cap B] + P[B\cap A^c]$$

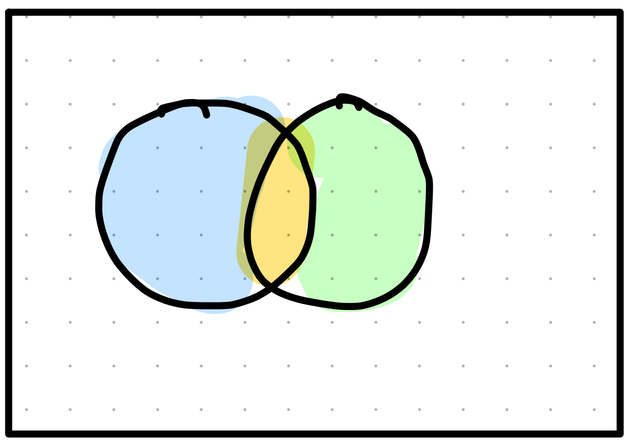
$$= P[A\cap B^c] + P[A\cap B] + P[B\cap A^c] + P[B\cap A] - P[A\cap B]$$

For any A and B

$$P[A\setminus B] + P[A\cap B] + P[B\setminus A]$$

$$= P[A\cap B^c] + P[A\cap B] + P[B\cap A^c]$$

$$= P[A\cap B^c] + P[A\cap B] + P[B\cap A^c] + P[B\cap A] - P[A\cap B]$$



$A\cap B^c$ Non overlapping

$A\cap B$

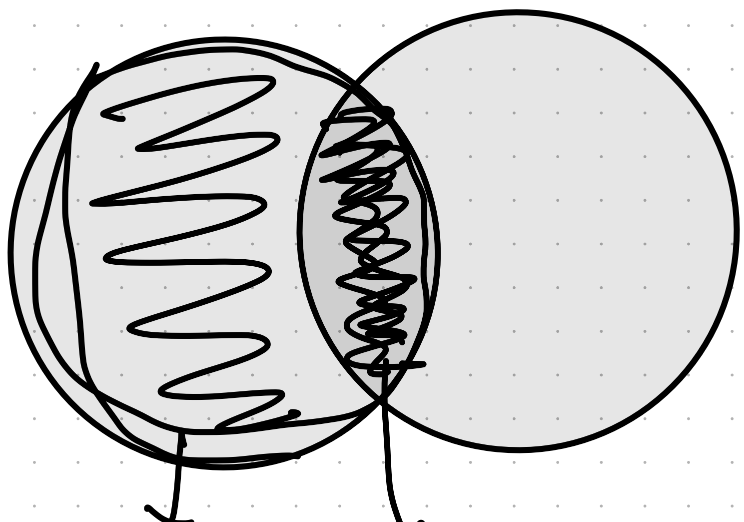
For any A and B

$$P[A\setminus B] + P[A\cap B] + P[B\setminus A]$$

$$= P[A] - P[A\cap B]$$

$$P[B\setminus A] \rightarrow P[B] - P[A\cap B]$$

A



B

A

$A\cap B$

$$P[A\setminus B] + P[A\cap B]$$
$$= P[A]$$

$$= P[A] + P[B] - P[A\cap B]$$

For any A and B

$$P[A\setminus B] + P[A\cap B] + P[B\setminus A]$$

$$= P[A\cap B^c] + P[A\cap B] + P[B\cap A^c]$$

$$= P[A\cap B^c] + P[A\cap B] + P[B\cap A^c] + P[B\cap A] - P[A\cap B]$$

$$= P[(A\cap B^c)\cup(A\cap B)] + P[(B\cap A^c)\cup(B\cap A)] - P(A\cap B)$$

For any A and B

$$P[A\setminus B] + P[A\cap B] + P[B\setminus A]$$

$$= P[A\cap B^c] + P[A\cap B] + P[B\cap A^c]$$

$$= P[A\cap B^c] + P[A\cap B] + P[B\cap A^c] + P[B\cap A] - P[A\cap B]$$

$$= P[(A\cap B^c)\cup(A\cap B)] + P[(B\cap A^c)\cup(B\cap A)] - P(A\cap B)$$

$$= P[A] + P[B] - P[A\cap B]$$

Q) Prove

$$P[A \cup B] \leq P[A] + P[B]$$

Q) Prove

$$P[A \cup B] \leq P[A] + P[B]$$

$$P[A \cup B] = P[A] + P[B] - \underbrace{P[A \cap B]}_{\geq 0}$$

$$\therefore \quad P[A \cup B] \leq P[A] + P[B]$$

$P(A) = x$; $P(B) = y$; $P(A \cup B) = z$

1) $P[A \cap B] = ?$

$P(A) = x; \quad P(B) = y; \quad P(A \cup B) = z$

i) $P[A \cap B] = ?$

$$P[A \cup B] = P[A] + P[B] - P[A \cap B]$$

$$\therefore \quad P[A \cap B] = P[A] + P[B] - P[A \cup B]$$

$$= x + y \cdot z$$

$P(A) = x$; $P(B) = y$; $P(A \cup B) = z$

2) $P[A^c \cap B^c] = ?$

$P(A) = x$; $P(B) = y$; $P(A \cup B) = z$

2) $P[A^c \cap B^c] = ?$

$= P[(A \cup B)^c]$

$= 1 - z$

$P(A) = x$; $P(B) = y$; $P(A \cup B) = z$

3) $P[A \cap B^c] = ?$

$P(A) = x; \quad P(B) = y; \quad P(A \cup B) = z$

3) $P[A \cap B^c] = ?$

$$= P[A \setminus B]$$

$$= P[A] - P[A \cap B]$$

$$= x - (x + y - z)$$

$$= z - y$$