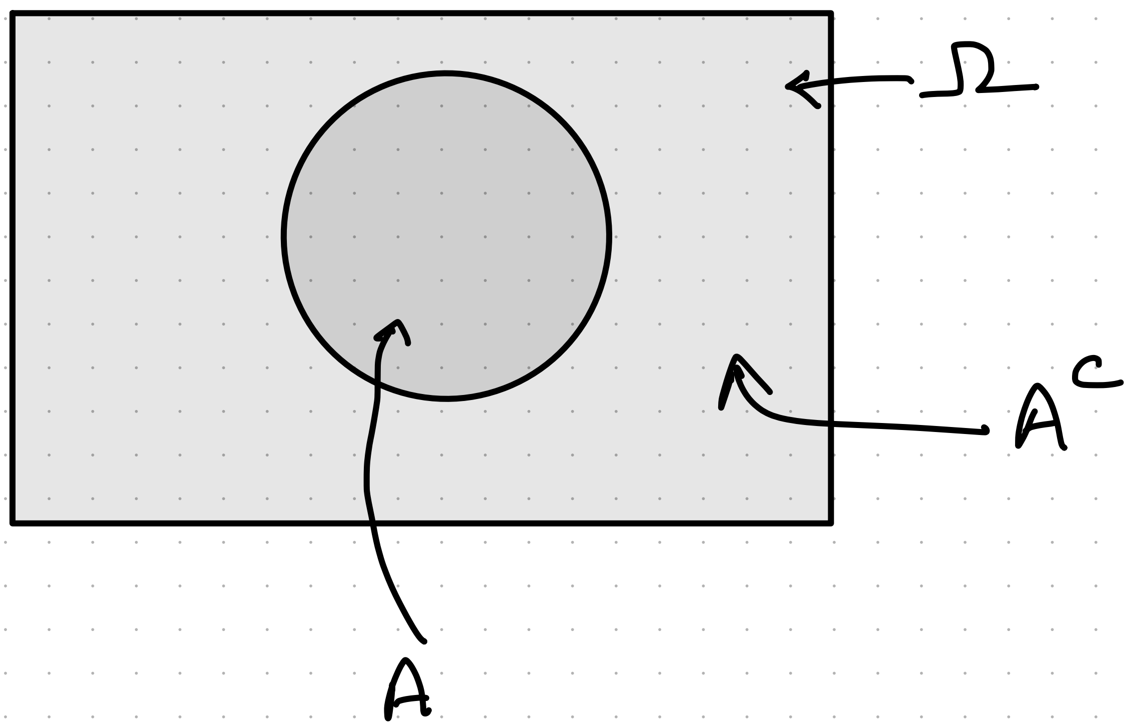
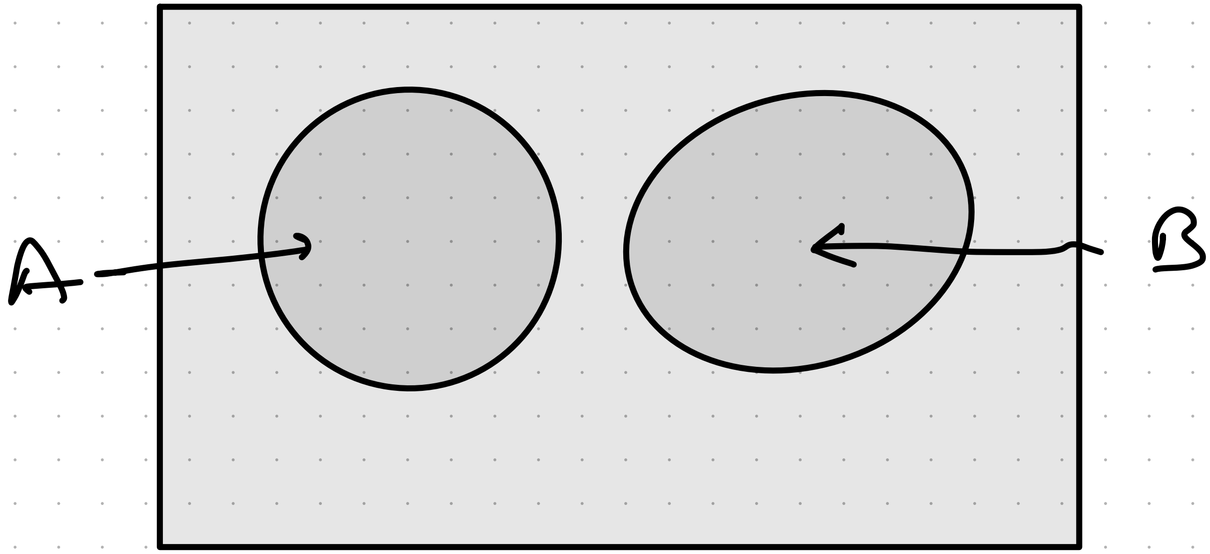
Complement



$$A^c = \Omega \setminus A$$

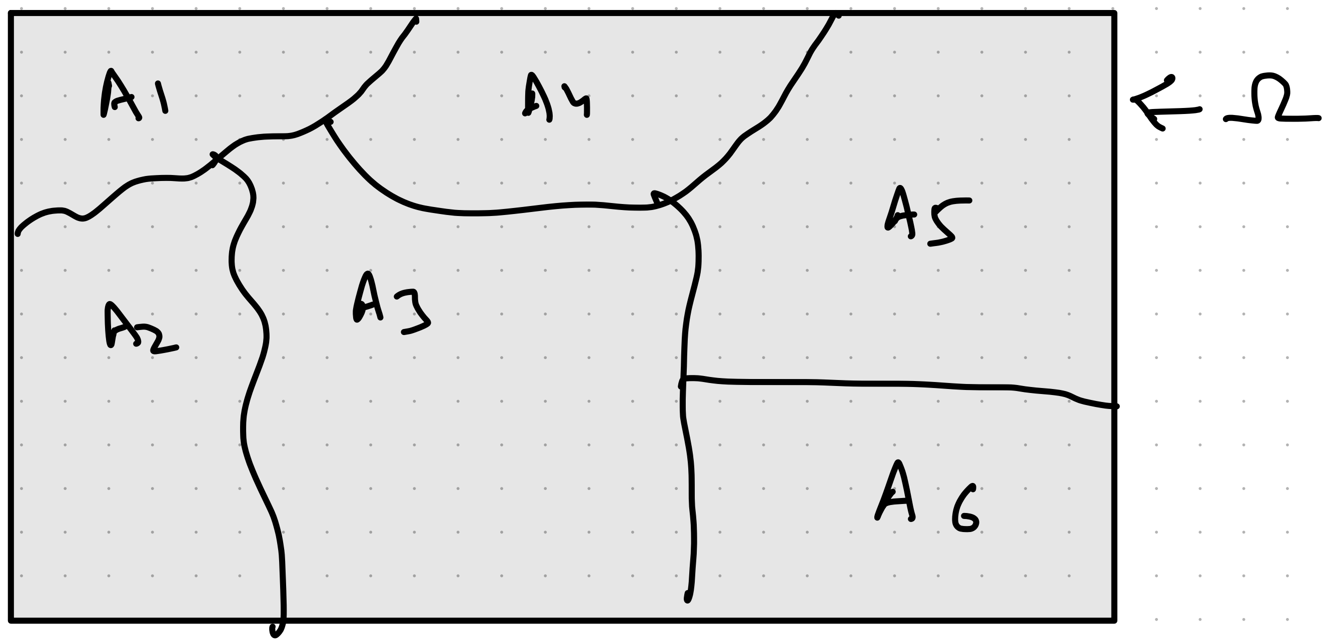


$$A \cap B = \phi$$

$$A = \{1, 3, 5\}$$

$$B = \{2, 4, 6\}$$

Partition



$$A_1 \cap A_2 = \phi$$

1) $A_i \cap A_j = \phi$

2) $A_1 \cup A_2 \cup \dots \, A_6 = \Omega$

Proof strategy

$$A = B$$

$$\left.\begin{array}{l} A \subseteq B \\ B \subseteq A \end{array}\right\} \Rightarrow A = B$$

Q) Prove if $A \subseteq B$, then $A \cup B = B$

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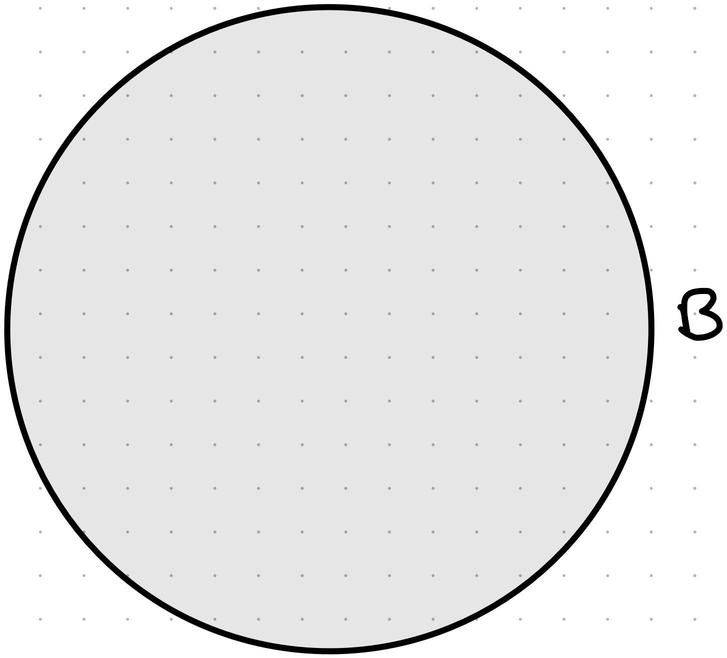
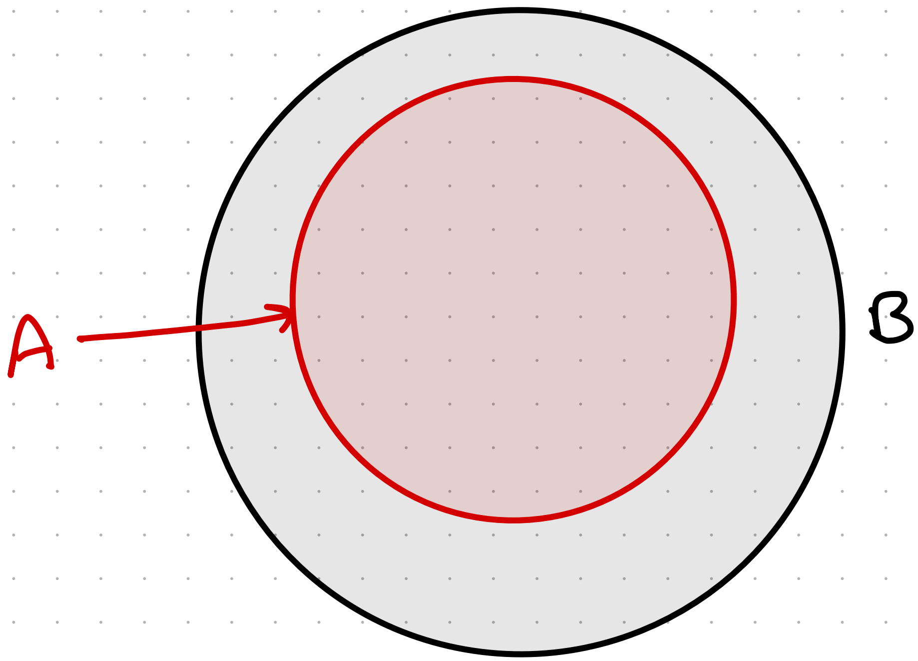
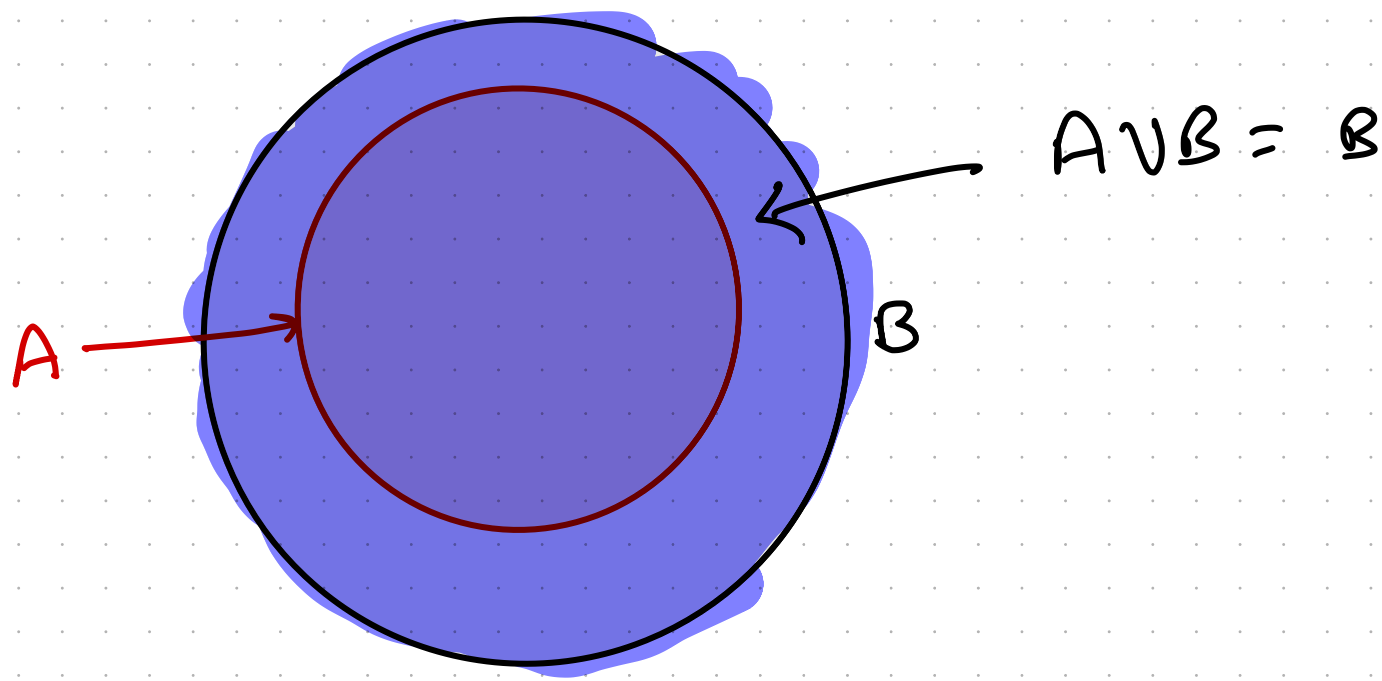


Q) Prove if $A \subseteq B$, then $A \cup B = B$



Q) Prove if $A \subseteq B$, then $A \cup B = B$



Q) Prove if $A \subseteq B$, then $A \cup B = B$

Start with $A \subseteq B$

$\Rightarrow x \in A \Rightarrow x \in B$

Q) Prove if $A \subseteq B$, then $A \cup B = B$

Start with $A \subseteq B$

$\Rightarrow x \in A \Rightarrow x \in B$

If we show $A \cup B \subseteq B$ and $B \subseteq A \cup B$

we are done

Q) Prove if $A \subseteq B$, then $A \cup B = B$

Start with $A \subseteq B$

$\Rightarrow x \in A \Rightarrow x \in B$

Let us work on: $A \cup B \subseteq B$

Q) Prove if $A \subseteq B$, then $A \cup B = B$

Start with $A \subseteq B$

$\Rightarrow x \in A \Rightarrow x \in B$

Let us work on: $A \cup B \subseteq B$

Let $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$

Q) Prove if $A \subseteq B$, then $A \cup B = B$

Start with $A \subseteq B$

$\Rightarrow x \in A \Rightarrow x \in B \quad \ldots (1)$

Let us work on: $A \cup B \subseteq B$

Let $x \in A \cup B$

$\Rightarrow x \in A \quad$ or $\quad x \in B$

If $x \in A \Rightarrow x \in B$ (From (1))

Q) Prove if $A \subseteq B$, then $A \cup B = B$

Start with $A \subseteq B$

$\Rightarrow x \in A \Rightarrow x \in B$ ... (1)

Let us work on: $A \cup B \subseteq B$

Let $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$

If $x \in A \Rightarrow x \in B$ (From (1))

Elif $x \in B$

Q) Prove if $A \subseteq B$, then $A \cup B = B$

Start with $A \subseteq B$

$\Rightarrow x \in A \Rightarrow x \in B \quad \ldots (1)$

Let us work on: $A \cup B \subseteq B$

Let $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$

If $x \in A \Rightarrow x \in B$ (From (1))

Elif $x \in B$

$\therefore$ Either way $x \in B$; or If $x \in A \cup B \Rightarrow x \in B$

or $A \cup B \subseteq B$

Q) Prove if $A \subseteq B$, then $A \cup B = B$

Start with $A \subseteq B$

$\Rightarrow \; x \in A \Rightarrow x \in B \; \ldots (1)$

Now let us work on: $B \subseteq A \cup B$

Q) Prove if $A \subseteq B$, then $A \cup B = B$

Start with $A \subseteq B$

$\Rightarrow x \in A \Rightarrow x \in B$ ... (1)

Now let us work on: $B \subseteq A \cup B$

let $x \in B$

Now $A \cup B = \{x \mid x \in A \text{ OR } x \in B\}$

Since $x \in B \quad \therefore x \in A \cup B$

$\therefore B \subseteq A \cup B$

Q) Prove if $A \subseteq B$, then $A \cup B = B$

Start with $A \subseteq B$

$\Rightarrow x \in A \Rightarrow x \in B$ ... 1

$\boxed{B \subseteq A \cup B}$ $\boxed{A \cup B \subseteq B}$

$\therefore$

$A \cup B = B$

Q) Prove if $A \subseteq B$; then $A \cap B = A$

Q) Prove if $A \subseteq B$; then $A \cap B = A$

$A \subseteq B$; If $x \in A \Rightarrow x \in B$ ... (1)

(I) To prove $A \cap B \subseteq A$

Let $x \in A \cap B$ then $x \in A$ AND $x \in B$

Since $x \in A \Rightarrow A \cap B \subseteq A$

(II) To prove $A \subseteq A \cap B$

Let $x \in A$; FROM (1) we know $x \in B$

$\Rightarrow x \in A$ AND $x \in B$ $\Rightarrow x \in A \cap B$

$\Rightarrow A \subseteq A \cap B$

$\therefore A \cap B \subseteq A$ AND $A \subseteq A \cap B$

$\Rightarrow A \cap B = A$

Q) Prove that $A \cup (B \cup C) = (A \cup B) \cup C$

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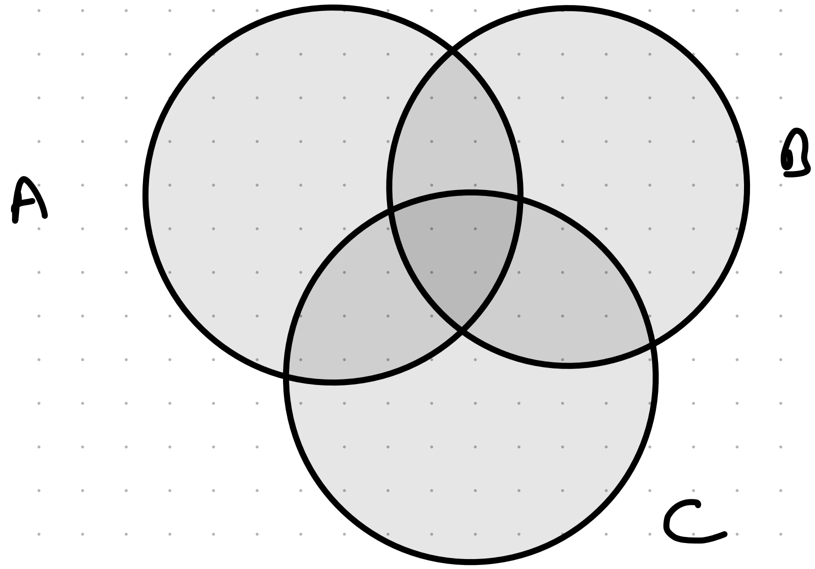
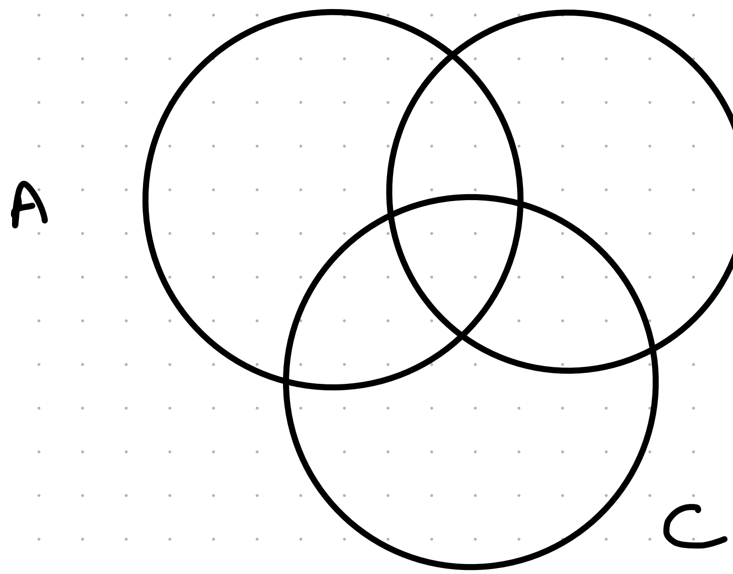
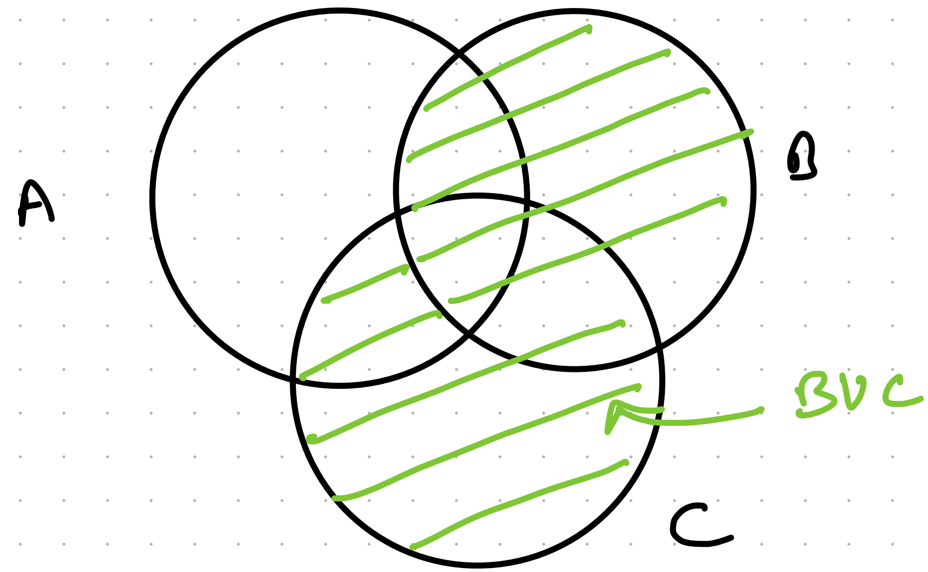
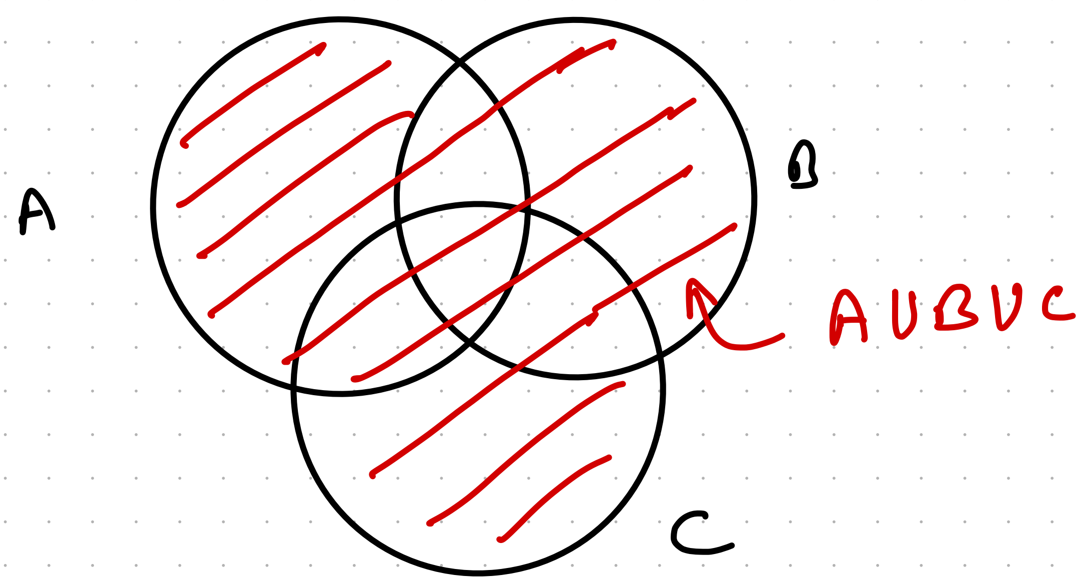


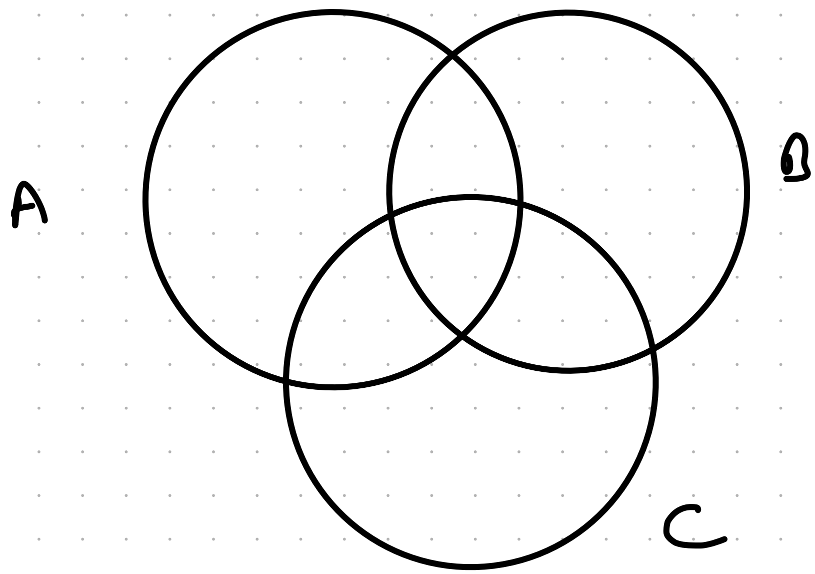
Q) Prove that $A \cup (B \cup C) = (A \cup B) \cup C$

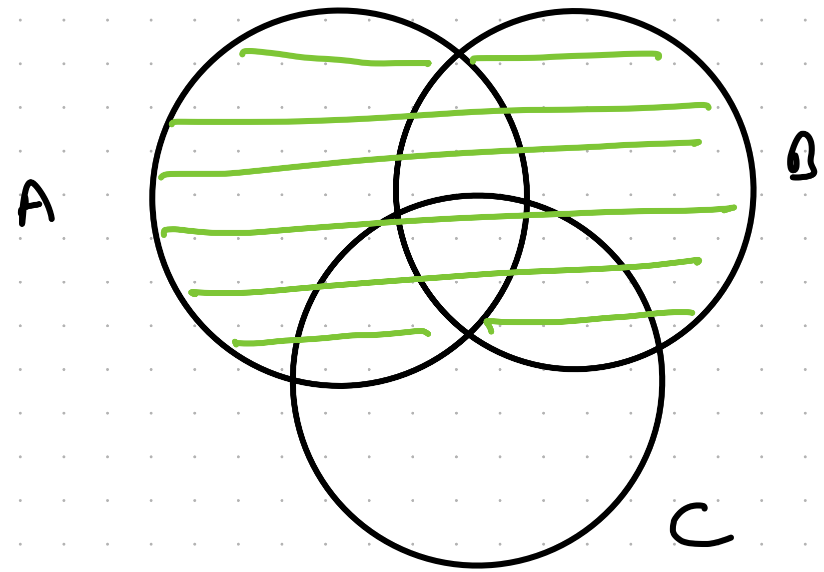


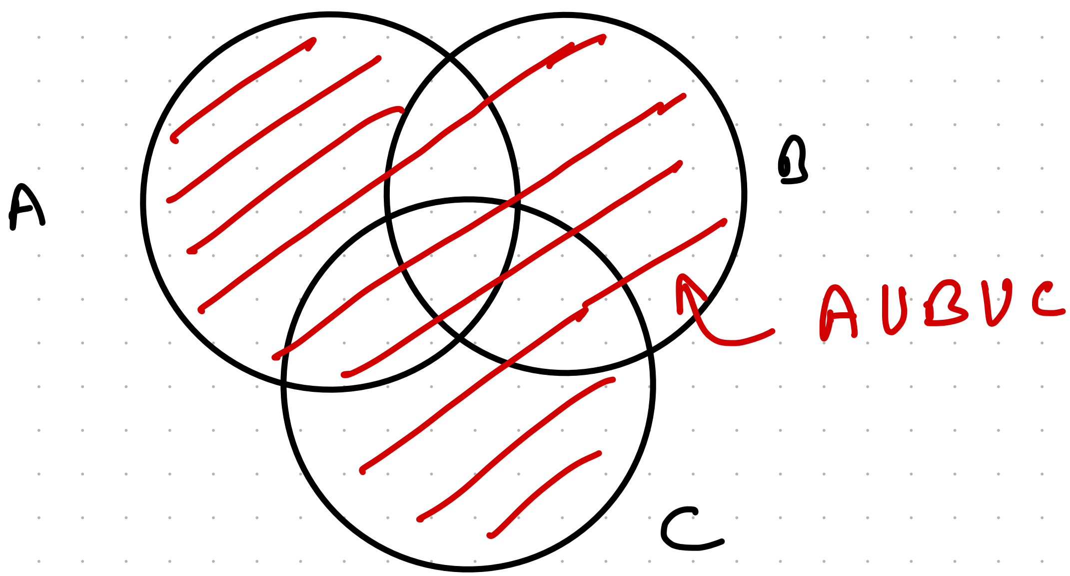




Q) Prove that $A \cup (B \cup C) = (A \cup B) \cup C$







Q) Prove that $A \cup (B \cup C) = (A \cup B) \cup C$

To prove

$$A \cup (B \cup C) \subseteq (A \cup B) \cup C$$

AND

$$(A \cup B) \cup C \subseteq A \cup (B \cup C)$$

Q) Prove that $A \cup (B \cup C) = (A \cup B) \cup C$

To prove

$$A \cup (B \cup C) \subseteq (A \cup B) \cup C$$

Let $x \in A \cup (B \cup C)$

$\Rightarrow x \in A$ or $x \in (B \cup C)$

Q) Prove that $A \cup (B \cup C) = (A \cup B) \cup C$

To prove

$$A \cup (B \cup C) \subseteq (A \cup B) \cup C$$

Let $x \in A \cup (B \cup C)$

$\Rightarrow x \in A$ or $x \in (B \cup C)$

Q) Prove that $A \cup (B \cup C) = (A \cup B) \cup C$

To prove

$$A \cup (B \cup C) \subseteq (A \cup B) \cup C$$

Let $x \in A \cup (B \cup C)$

$\Rightarrow x \in A$ or $x \in (B \cup C)$

$\Downarrow$

$x \in (A \cup B)$

$\Downarrow$

$x \in (A \cup B) \cup C$

Q) Prove that $A \cup (B \cup C) = (A \cup B) \cup C$

To prove

$$A \cup (B \cup C) \subseteq (A \cup B) \cup C$$

Let $x \in A \cup (B \cup C)$

$\Rightarrow x \in A$ or $x \in (B \cup C)$

$\Rightarrow x \in B$ or $x \in C$

$\Downarrow$

$x \in (A \cup B)$

$\Downarrow$

$x \in (A \cup B) \cup C$

Q) Prove that $A \cup (B \cup C) = (A \cup B) \cup C$

To prove

$$A \cup (B \cup C) \subseteq (A \cup B) \cup C$$

Let $x \in A \cup (B \cup C)$

$\Rightarrow x \in A$ or $x \in (B \cup C)$

$x \in A$

$\Downarrow$

$x \in (A \cup B)$

$\Downarrow$

$x \in (A \cup B) \cup C$

$x \in (B \cup C)$

$\Rightarrow x \in B$ or $x \in C$

$x \in B$

$\Downarrow$

$x \in (A \cup B)$

$\Downarrow$

$x \in (A \cup B) \cup C$

Q) Prove that $A \cup (B \cup C) = (A \cup B) \cup C$

To prove

$$A \cup (B \cup C) \subseteq (A \cup B) \cup C$$

Let $x \in A \cup (B \cup C)$

$\Rightarrow x \in A$ or $x \in (B \cup C)$

$\Rightarrow x \in B$ or $x \in C$

$x \in A \Rightarrow x \in (A \cup B) \Rightarrow x \in (A \cup B) \cup C$

$x \in B \Rightarrow x \in (A \cup B) \Rightarrow x \in (A \cup B) \cup C$

$x \in C \Rightarrow x \in C \cup (A \cup B) \Rightarrow x \in (A \cup B) \cup C$

Q) Prove that $A \cup (B \cup C) = (A \cup B) \cup C$

To prove

$$A \cup (B \cup C) \subseteq (A \cup B) \cup C$$

Let $x \in A \cup (B \cup C)$

$\Rightarrow x \in A$ or $x \in (B \cup C)$

$\Rightarrow x \in B$ or $x \in C$

Case $x \in A$:

$$\Downarrow$$

$$x \in (A \cup B)$$

$$\Downarrow$$

$$x \in (A \cup B) \cup C$$

Case $x \in B$:

$$\Downarrow$$

$$x \in (A \cup B)$$

$$\Downarrow$$

$$x \in (A \cup B) \cup C$$

Case $x \in C$:

$$\Downarrow$$

$$x \in C \cup (A \cup B)$$

$$\Downarrow$$

$$x \in (A \cup B) \cup C$$

$$\therefore A \cup (B \cup C) \subseteq (A \cup B) \cup C$$

Q) Prove that $A \cup (B \cup C) = (A \cup B) \cup C$

To prove

$$(A \cup B) \cup C \subseteq A \cup (B \cup C)$$

Let $x \in (A \cup B) \cup C$

$\Rightarrow x \in A \cup B$ or $x \in C$

$\Rightarrow x \in A$ OR $x \in B$ OR $x \in C$

Q) Prove that $A \cup (B \cup C) = (A \cup B) \cup C$

To prove

$$(A \cup B) \cup C \subseteq A \cup (B \cup C)$$

Let $x \in (A \cup B) \cup C$

$\Rightarrow x \in A \cup B$ or $x \in C$

$\Rightarrow x \in A$ OR $x \in B$ OR $x \in C$

$\Downarrow$ $\Downarrow$ $\Downarrow$

$x \in A \cup (B \cup C)$ | $x \in B \cup C$ | $x \in B \cup C$

$\Downarrow$ $\Downarrow$

$x \in A \cup (B \cup C)$ | $x \in A \cup (B \cup C)$

$$\therefore (A \cup B) \cup C \subseteq A \cup (B \cup C)$$

Q) Prove that $A \cup (B \cup C) = (A \cup B) \cup C$

$$(A \cup B) \cup C \subseteq A \cup (B \cup C)$$

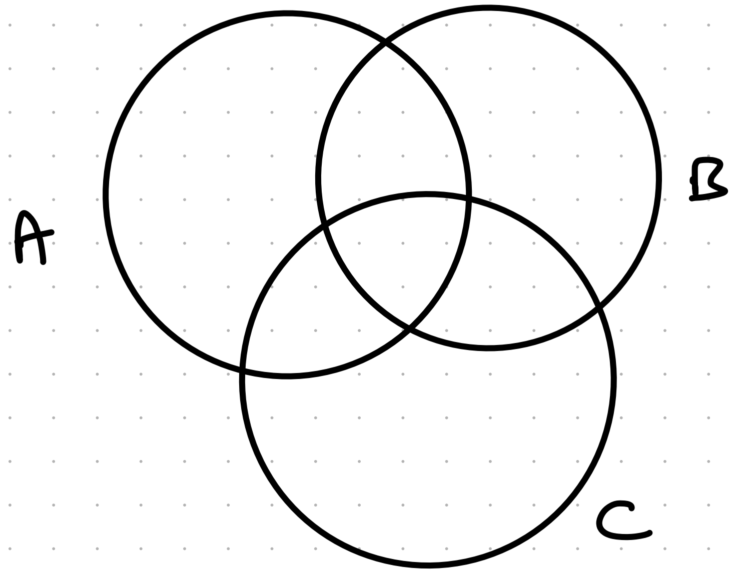
AND

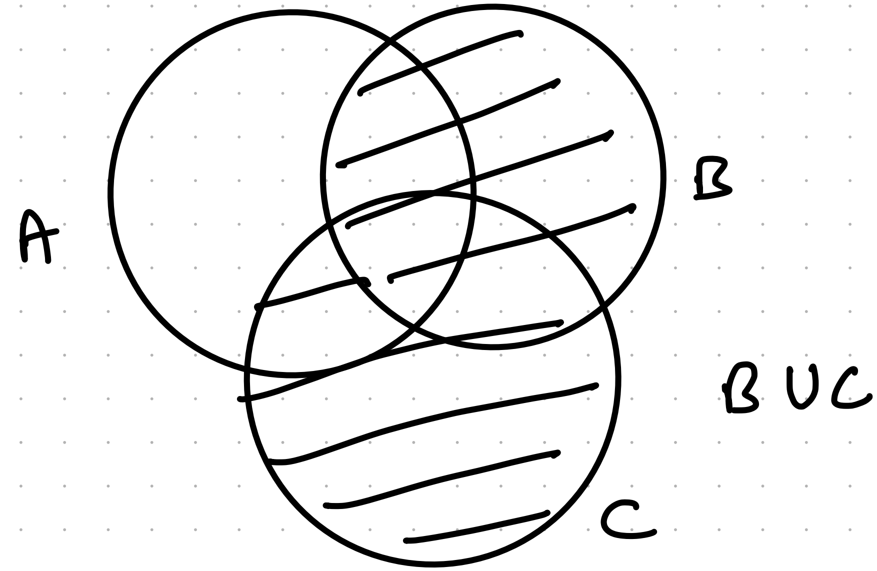
$$A \cup (B \cup C) \subseteq (A \cup B) \cup C$$

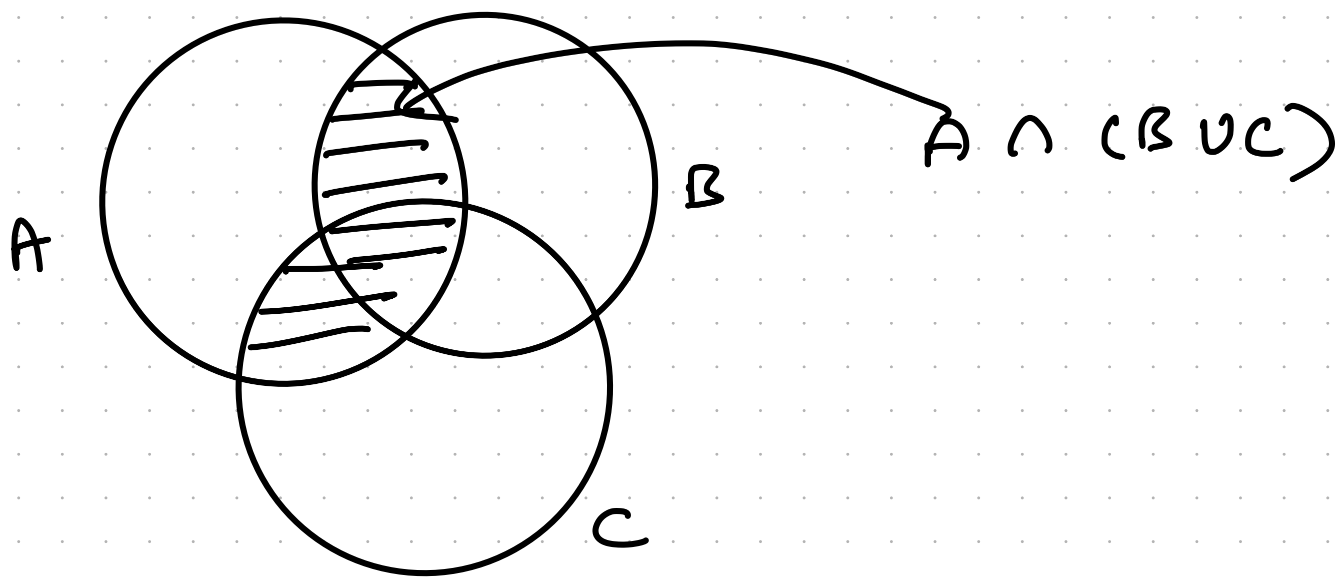
$$\therefore \quad A \cup (B \cup C) = (A \cup B) \cup C$$

Q) Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

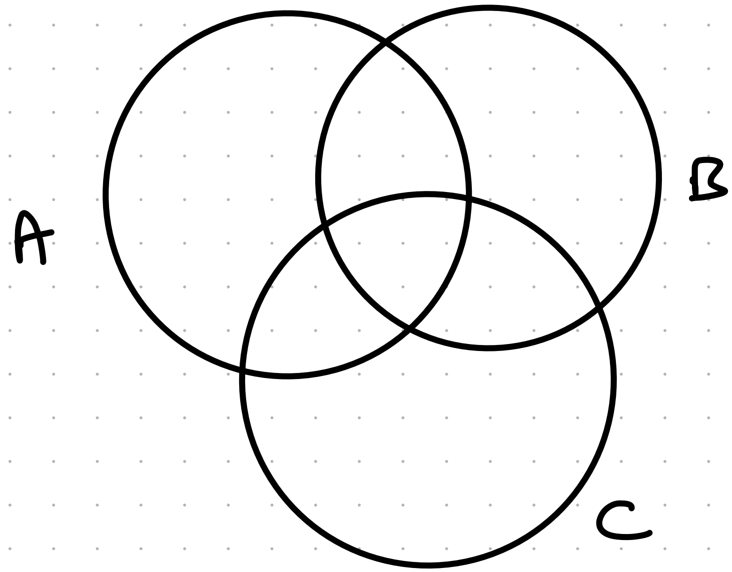
Q) Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

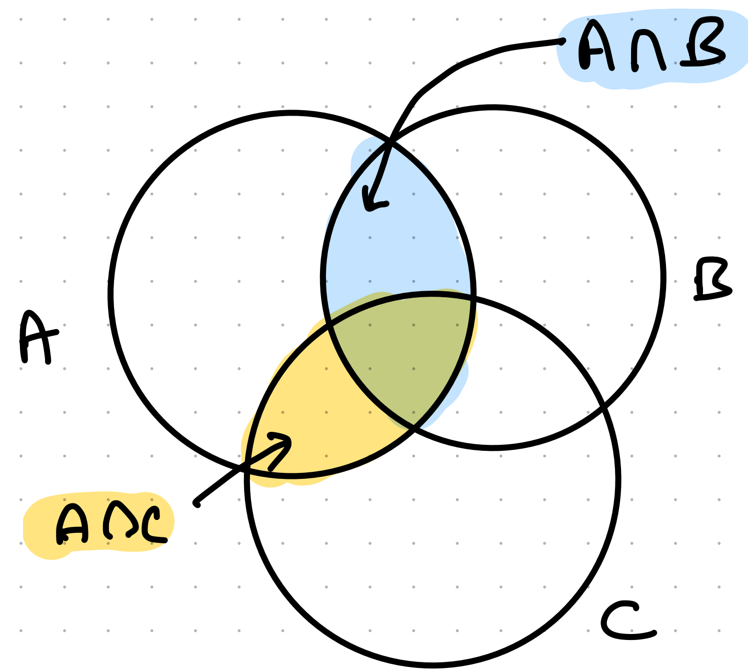


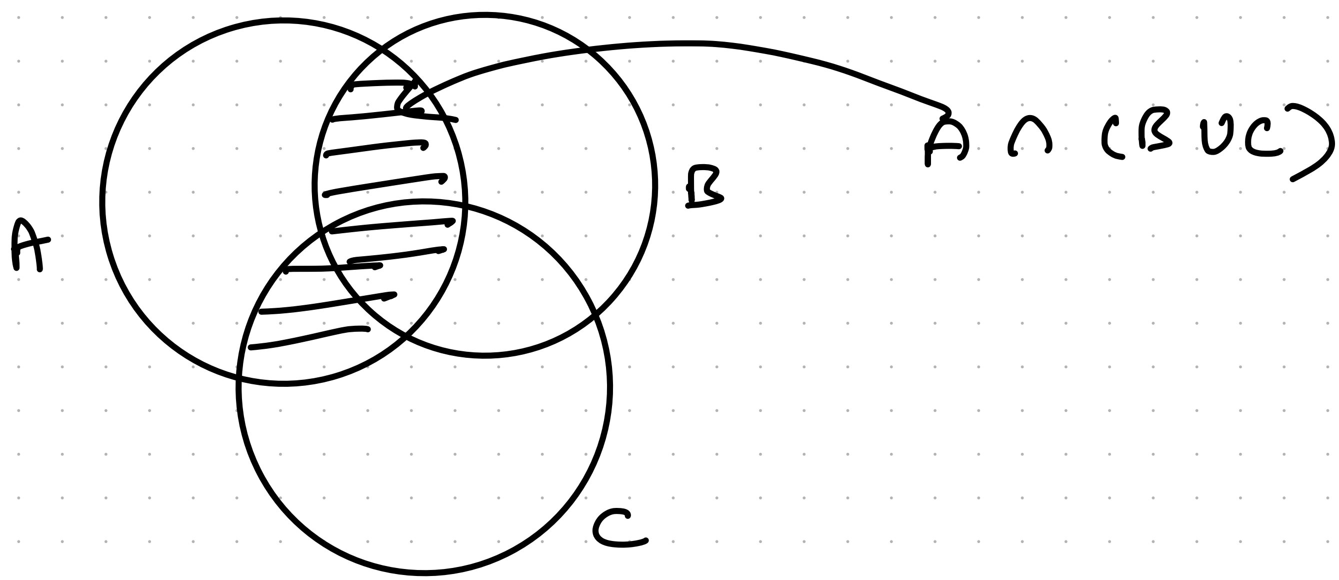




Q) Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$







Q) Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

We need to show

$$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C) \quad \text{and}$$

$$(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$$

Q) Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

We need to show

$$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$$

Let $x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ AND $[x \in B$ or $x \in C]$

Q) Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

We need to show

$$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$$

Let $x \in A \cap (B \cup C)$

$\Rightarrow$ $x \in A$ AND $[x \in B$ or $x \in C]$

$$\Downarrow$$

If $x \in B$

Q) Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

We need to show

$$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$$

Let $x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ AND $[x \in B$ or $x \in C]$

$\Downarrow$

If $x \in B$

$\Downarrow$

$x \in A \cap B$

Q) Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

We need to show

$$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$$

Let $x \in A \cap (B \cup C)$

$\Rightarrow$ $x \in A$ AND $[x \in B$ or $x \in C]$

$\Downarrow$ $\Downarrow$

If $x \in B$ If $x \in C$

$\Downarrow$

$x \in A \cap C$

Q) Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

We need to show

$$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$$

Let $x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ AND $[x \in B$ or $x \in C]$

$\Downarrow$ $\Downarrow$

If $x \in B$ If $x \in C$

$\Downarrow$

$x \in A \cap C$

If $x \in B \Rightarrow x \in A \cap B$

OR If $x \in C \Rightarrow x \in A \cap C$

$\therefore \; x \in (A \cap B) \cup (A \cap C)$

$$\Rightarrow \boxed{A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)}$$

Q) Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

We need to show $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$

Let $x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow$ $x \in A \cap B$ OR $x \in A \cap C$

$\Rightarrow$ $x \in A$ AND $x \in B$ OR $x \in A$ AND $x \in C$

Q) Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

We need to show $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$

Let $x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow x \in A \cap B$ OR $x \in A \cap C$

$\Rightarrow x \in A$ AND $x \in B$ OR $x \in A$ AND $x \in C$

Both cases $x \in A$

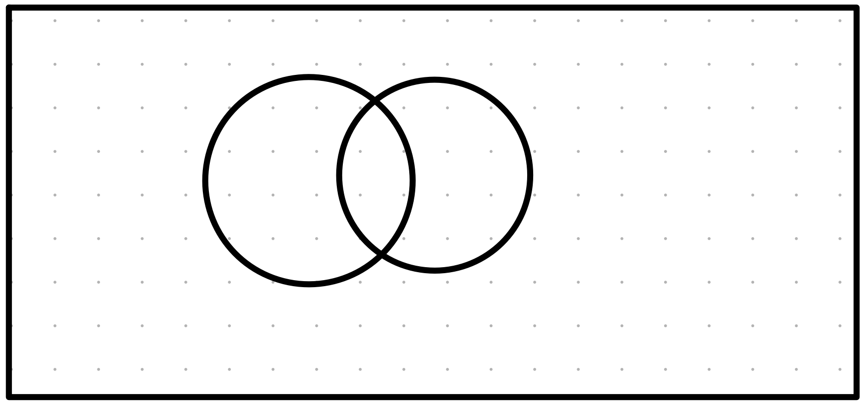
$\therefore x \in A$ AND $x \in B$ or $x \in C$

$\therefore x \in A \cap (B \cup C)$

$\therefore (A \cup B) \cup (A \cap C) \subseteq A \cap (B \cup C)$

Thus; $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Q) Prove $(A \cap B)^c = A^c \cup B^c$



Q) Prove $(A \cap B)^c = A^c \cup B^c$

I) $(A \cap B)^c \subseteq A^c \cup B^c$

II) $A^c \cup B^c \subseteq (A \cap B)^c$

Q) Prove $(A \cap B)^c = A^c \cup B^c$

I) $(A \cap B)^c \subseteq A^c \cup B^c$

Let $x \in (A \cap B)^c$

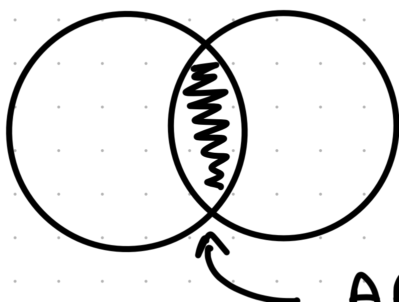
$\Rightarrow x \notin A \cap B$

Q) Prove $(A \cap B)^c = A^c \cup B^c$

I) $(A \cap B)^c \subseteq A^c \cup B^c$

Let $x \in (A \cap B)^c$

$\Rightarrow x \notin A \cap B$



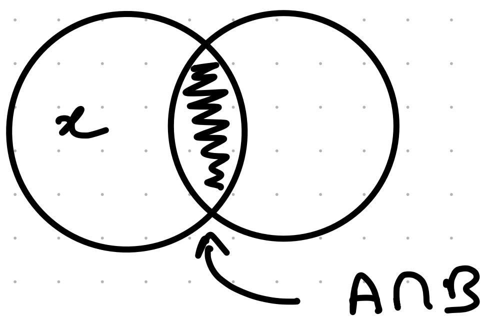
$A \cap B$

Q) Prove $(A \cap B)^c = A^c \cup B^c$

I) $(A \cap B)^c \subseteq A^c \cup B^c$

Let $x \in (A \cap B)^c$

$\Rightarrow x \notin A \cap B$



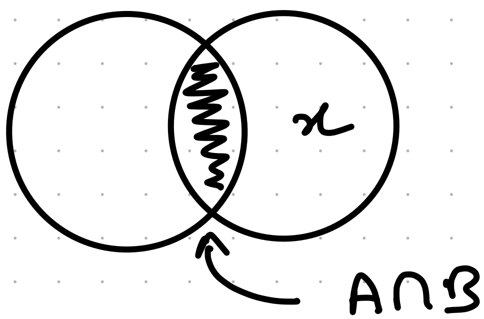
Here $x \notin B \Rightarrow x \in B^c$

Q) Prove $(A \cap B)^c = A^c \cup B^c$

I) $(A \cap B)^c \subseteq A^c \cup B^c$

Let $x \in (A \cap B)^c$

$\Rightarrow x \notin A \cap B$



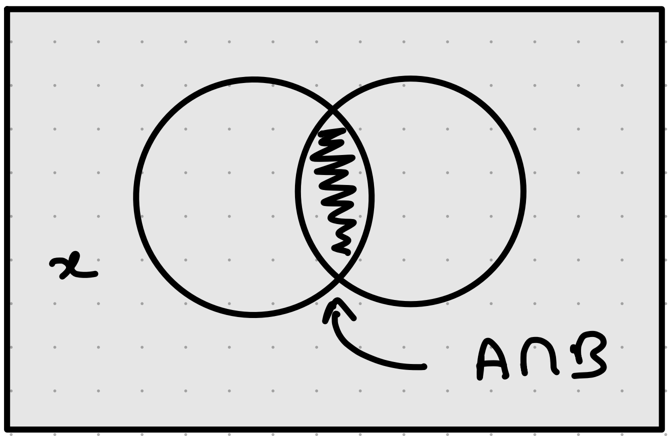
Here $x \notin A \Rightarrow x \in A^c$

Q) Prove $(A \cap B)^c = A^c \cup B^c$

I) $(A \cap B)^c \subseteq A^c \cup B^c$

Let $x \in (A \cap B)^c$

$\Rightarrow x \notin A \cap B$



Here $x \in A^c$ and $x \in B^c$

Q) Prove $(A \cap B)^c = A^c \cup B^c$

I) $(A \cap B)^c \subseteq A^c \cup B^c$

Let $x \in (A \cap B)^c$

$\Rightarrow x \notin A \cap B$

If $x \notin A$

$\Downarrow$

$x \in A^c$

If $x \notin B$

$\Downarrow$

$x \in B^c$

Q) Prove $(A \cap B)^c = A^c \cup B^c$

I) $(A \cap B)^c \subseteq A^c \cup B^c$

Let $x \in (A \cap B)^c$

$\Rightarrow x \notin A \cap B$

If $x \notin A$ $\Downarrow$ $x \in A^c$

If $x \notin B$ $\Downarrow$ $x \in B^c$

$\Rightarrow x \in A^c$ or $x \in B^c$

$\Rightarrow x \in A^c \cup B^c$

$\Rightarrow (A \cap B)^c \subseteq A^c \cup B^c$

Q) Prove $(A \cap B)^c = A^c \cup B^c$

I) $(A \cap B)^c \subseteq A^c \cup B^c$

Let $x \in (A \cap B)^c$

$\Rightarrow x \notin A \cap B$

If $x \notin A$

$\Downarrow$

$x \in A^c$

If $x \notin B$

$\Downarrow$

$x \in B^c$

$\Rightarrow x \in A^c$ or $x \in B^c$

$\Rightarrow x \in A^c \cup B^c$

$\Rightarrow (A \cap B)^c \subseteq A^c \cup B^c$

$x \notin A \cap B$ means:

1) x is not in set of elements that belong to both A and B

2) x fails to meet atleast

→ x is not in A

OR

→ x is not in B

OR

BOTH

Q) Prove $(A \cap B)^c = A^c \cup B^c$

I) $(A \cap B)^c \subseteq A^c \cup B^c$

let $x \in (A \cap B)^c$

$\Rightarrow x \notin A \cap B$

If $x \notin A$

$\Downarrow$

$x \in A^c$

If $x \notin B$

$\Downarrow$

$x \in B^c$

$\Rightarrow x \in A^c$ or $x \in B^c$

$\Rightarrow x \in A^c \cup B^c$

$\Rightarrow (A \cap B)^c \subseteq A^c \cup B^c$

Example

$A = \{1, 2, 3\}$

$B = \{2, 3, 4\}$

$\Omega = \{1, 2, 3, 4, 5\}$

$x \in (A \cap B)^c$

$x \in \{1, 4, 5\}$

Consider $x = 1$

$x = 1 \in A$; $x = 1 \in B^c$

$x = 4$

$x = 4 \in B$; $x = 4 \in A^c$

$x = 5 \in A^c$; $x = 5 \in B^c$

$\therefore x \in A^c$ or $x \in B^c$

Q) Prove $(A \cap B)^c = A^c \cup B^c$

I) $(A \cap B)^c \subseteq A^c \cup B^c$

Let $x \in (A \cap B)^c$

$\Rightarrow x \notin A \cap B$

If $x \notin A$ $\Downarrow$ $x \in A^c$

If $x \notin B$ $\Downarrow$ $x \in B^c$

$\Rightarrow x \in A^c$ or $x \in B^c$

$\Rightarrow x \in A^c \cup B^c$

$\Rightarrow (A \cap B)^c \subseteq A^c \cup B^c$

<u>Logic</u>

$x \in (A \cap B)^c$

$\Rightarrow$ NOT (x in A and B)

$\Rightarrow$ NOT(x in A) OR NOT(x in B)

$\Rightarrow$ x in A^c OR x in B^c

Logical negation of AND becomes OR

Logical Negation

$$\neg (P \vee Q) \Rightarrow \neg P \wedge \neg Q$$

$$\neg (p \wedge q) \Rightarrow \neg p \vee \neg q$$

P and Q are statements : True or False

Logical OR

Logical Negation

Logical AND

$$\neg (P \lor Q) \Rightarrow \neg P \land \neg Q$$

P and Q are statements : True or False

p	q	$p \lor q$	$\neg(p \lor q)$	$\neg p$	$\neg q$	$\neg p \land \neg q$
F	F	F	T	T	T	T
F	T	T	F	T	F	F
T	F	T	F	F	T	F
T	T	T	F	F	F	F

Q) Prove $(A \cap B)^c = A^c \cup B^c$

I) $A^c \cup B^c \subseteq (A \cap B)^c$

Q) Prove $(A \cap B)^c = A^c \cup B^c$

II) $A^c \cup B^c \subseteq (A \cap B)^c$

Let $x \in A^c \cup B^c$

$\Rightarrow x \in A^c$ OR $x \in B^c$

Q) Prove $(A \cap B)^c = A^c \cup B^c$

II) $A^c \cup B^c \subseteq (A \cap B)^c$

Let $x \in A^c \cup B^c$

$\Rightarrow x \in A^c$ OR $x \in B^c$

$\Downarrow$ $\Downarrow$

$x \notin A$ $x \notin B$

$x \notin A \cap B$

$\Rightarrow x \in (A \cap B)^c$

$\therefore A^c \cup B^c \subseteq (A \cap B)^c$

$\therefore (A \cap B)^c = A^c \cup B^c$

Q) Prove $(A \cap B)^c = A^c \cup B^c$

II) $A^c \cup B^c \subseteq (A \cap B)^c$

Let $x \in A^c \cup B^c$

$\Rightarrow x \in A^c$ OR $x \in B^c$

$\Downarrow$ $\Downarrow$

$x \notin A$ $x \notin B$

Case 1) If $x \notin A$; x cannot be in $A \cap B$

$\Rightarrow x \notin A \cap B$

Q) Prove $(A \cap B)^c = A^c \cup B^c$

II) $A^c \cup B^c \subseteq (A \cap B)^c$

Let $x \in A^c \cup B^c$

$\Rightarrow x \in A^c$ OR $x \in B^c$

$\Downarrow$ $\Downarrow$

$x \notin A$ $x \notin B$

Case 1) If $x \notin A$; x cannot be in $A \cap B$

$\Rightarrow x \notin A \cap B$

Case 2) If $x \notin B$; x cannot be in $A \cap B$

$\Rightarrow x \notin A \cap B$

Thus either way

$x \notin A \cap B \Rightarrow x \in (A \cap B)^c$

Q) Prove $(A \cap B)^c = A^c \cup B^c$

II) $A^c \cup B^c \subseteq (A \cap B)^c$

Let $x \in A^c \cup B^c$

$\Rightarrow x \in A^c$ OR $x \in B^c$

$\Downarrow$ $\Downarrow$

$x \notin A$ $x \notin B$

eg. $A = \{1, 2, 3\}$; $B = \{2, 3, 4\}$; $A \cap B = \{2, 3\}$

Take $x = 1$; $x \notin B$ $\therefore x \notin A \cap B$

Take $x = 4$; $x \notin A$ $\therefore x \notin A \cap B$

Q) Prove $A \setminus B$ (A minus B) $= A \cap B^c$

Q) Prove $A \setminus B$ (A minus B) $= A \cap B^c$

Let $x \in A \setminus B$

$\Rightarrow x \in A$ and $x \notin B$

$\therefore x \in A$ and $x \in B^c$

$\Rightarrow x \in A \cap B^c \Rightarrow A \setminus B \subseteq A \cap B^c$

Q) Prove $A \setminus B$ (A minus B) $= A \cap B^c$

Let $x \in A \setminus B$

$\Rightarrow x \in A$ and $x \notin B$

$\therefore x \in A$ and $x \in B^c$

$\Rightarrow x \in A \cap B^c \Rightarrow A \setminus B \subseteq A \cap B^c$

Let $x \in A \cap B^c$

$\Rightarrow x \in A$ and $x \in B^c$

$\Rightarrow x \in A$ and $x \notin B$

$\Rightarrow x \in A \setminus B \Rightarrow A \cap B^c \subseteq A \setminus B$