MA 102: Linear Algebra

Quiz 2 — Determinants & Eigenvalues — ANSWER KEY

Duration: 30 minutes   Maximum marks: 12

SET B

Instructions

- Write your name and roll number before starting.
- This paper has 6 questions in 3 parts, for a total of 12 marks.

Part A — Multiple Choice

6 marks

Q1. Compute $\det\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$. *[2 marks]*

(A) 5 ✓
(B) -5
(C) 11
(D) 8

Explanation. $2 \cdot 4 - 1 \cdot 3 = 5$.

Q2. For a square real matrix A, which conditions are **equivalent** to invertibility? *(Select all that apply.)* *[2 marks]*

(A) **A has full rank** ✓
(B) $\det A \neq 0$ ✓
(C) A is diagonalizable
(D) A is symmetric

Explanation. Symmetry and diagonalizability are neither necessary nor sufficient: $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ is symmetric and diagonalizable but singular.

Q3. What is the rank of the 3×3 matrix of all ones, $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$? *[2 marks]*

(A) 3
(B) 2
(C) 0
(D) 1 ✓

Explanation. Every row is the same nonzero vector.

Part B — Fill in the Blanks

2 marks

Q4. A nonzero vector v is an eigenvector of A when $Av = \underline{\lambda}\, v$ for some scalar. *[1 mark]*

Q5. For square matrices, $\det(AB) = \det(A) \cdot \underline{\det(B)}$. *[1 mark]*

Part C — Short Answer

4 marks

Q6. Let λ be an eigenvalue of A with eigenvector v. Show that λ^2 is an eigenvalue of A^2, and state the corresponding eigenvector. Then give a 2×2 example where A^2 has an eigenvalue that A does *not* have… is that possible if we only square eigenvalues? Justify briefly. *[4 marks]*

Model answer. $A^2 v = A(Av) = A(\lambda v) = \lambda(Av) = \lambda^2 v$, so λ^2 is an eigenvalue with the **same** eigenvector v. Over $\mathbb{C}$ every eigenvalue of A^2 is the square of an eigenvalue of A (triangularize), so squaring cannot create eigenvalues unrelated to those of A.

Rubric. $+2$ the computation $A^2 v = \lambda^2 v$; $+1$ eigenvector is the same v; $+1$ justification that eigenvalues of A^2 are exactly squares of eigenvalues of A (over $\mathbb{C}$), so no new ones appear.

— *End of paper* —