MA 102: Linear Algebra

Quiz 2 — Determinants & Eigenvalues

Duration: 30 minutes   Maximum marks: 12

SET B

Name: ____________________ Roll No.: ____________

Instructions

- Write your name and roll number before starting.
- This paper has 6 questions in 3 parts, for a total of 12 marks.

Part A — Multiple Choice *6 marks*

Q1. Compute $\det\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$. *[2 marks]*

(A) 5
(B) -5
(C) 11
(D) 8

Q2. For a square real matrix A, which conditions are **equivalent** to invertibility? *(Select all that apply.)* *[2 marks]*

(A) A has full rank
(B) $\det A \neq 0$
(C) A is diagonalizable
(D) A is symmetric

Q3. What is the rank of the 3×3 matrix of all ones, $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$? *[2 marks]*

(A) 3
(B) 2
(C) 0
(D) 1

Part B — Fill in the Blanks *2 marks*

Q4. A nonzero vector v is an eigenvector of A when $Av =$ ________ v for some scalar. *[1 mark]*

Q5. For square matrices, $\det(AB) = \det(A) \cdot$ __________. *[1 mark]*

Part C — Short Answer *4 marks*

Q6. Let λ be an eigenvalue of A with eigenvector v. Show that λ^2 is an eigenvalue of A^2, and state the corresponding eigenvector. Then give a 2×2 example where A^2 has an eigenvalue that A does *not* have… is that possible if we only square eigenvalues? Justify briefly. *[4 marks]*

— *End of paper* —